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D.I. Bazov

HELICOPTER AERODYNAMICS

Translation of "Aerodinamika vertoletov."
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ANNOTATION

Principles of helicopter flight under various conditions are reviewed, giving special attention to the operation of the main rotor. A brief history of helicopter development is presented, together with a summary of the main components of a helicopter and a classification of the various types of helicopters. The characteristics of the main rotor and its operation during autorotation and during axial and oblique flow are considered. Also considered are vertical and horizontal flight, altitude gain and descent, takeoff and landing, equilibrium, stability, and controllability, taking into account the aerodynamic forces acting on the helicopter during the various maneuvers.

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CHAPTER I

PRINCIPLES OF HELICOPTER FLIGHT

§ 1. Brief History of Helicopter Development

The idea of creating a flying apparatus with an aerial screw, which created a lifting force, was suggested for the first time in 1475 by Leonardo de Vinci. This idea was too premature owing to the impossibility of technical realization of the project and opposition by religious opinions. The idea was buried in the archives. A sketch and description of this flying apparatus was displayed in the Milan library and published at the end of the 19th century. /3*

In 1754, M. V. Lomonosov substantiated the possibility of creating a heavier than air flying apparatus and built a model of a dual rotor helicopter with the rotors arranged coaxially.

In the 19th century many Russian scientists and engineers developed projects for flying machines with main rotors. In 1869, electrical engineer A. N. Lodygin proposed a projected helicopter powered by an electric motor.

In 1870 the well known scientist M. A. Rykachev was engaged in the development of propellers.

Metallurgist-scientist D. K. Chernov devised a helicopter scheme with longitudinal, transverse, and coaxially arranged rotors.

*Numbers in the margin indicate pagination in the original foreign text.

At the end of the 19th century, the development of flying machines engaged the attention of the distinguished Russian scientists D. I. Mendeleyev, K. E. Tsiolkovskiy, N. Ye. Zhukovskiy and S. A. Chaplygin. A period of indepth scientific substantiation of the idea of flight with heavier than air flying machines began.

A close associate of N. Ye. Zhukovskiy, B. N. Yur'yev, in 1911 proposed a well-developed single rotor helicopter project with a propeller for directional control and also a fundamental arrangement for helicopter control, that of automatically warping the main rotor. After the Great October Socialist Revolution, when our country began to develop its own aviation industry, work on the creation of a helicopter was continued.

In 1925, in TSAGI, an experimental group for special constructions was organized under the leadership of B. N. Yur'yev. This group was engaged in the development of a helicopter.

In 1930 the first Soviet helicopter was built, the TSAGI 1-EA (Figure 1). /4
This helicopter was tested by the engineer responsible for its construction, Aleksey Mikhaylovich Cheremyukhin. Cheremyukhin set a world record altitude of 605 m in this helicopter.

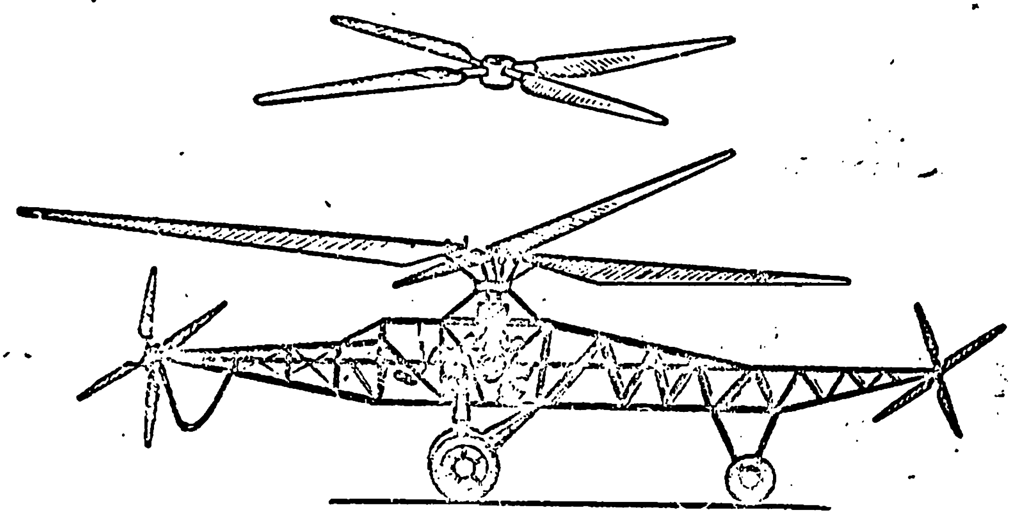

Figure 1. TSAGI 1-EA Helicopter.

In 1948 the single rotor helicopters Mi-1 and Yak-100 were built. As a result of the State trials, the helicopter Mi-1 proved to have the most satisfactory characteristics and it was accepted for mass production.

In 1952 the helicopter Mi-4 was built, which, for that time, had a very large useful load. The same year saw the completion and first flight of the tandem arrangement dual rotor helicopter, the Yak-24, "Flying Wagon" designed by A. S. Yakovlev (Figure 2).

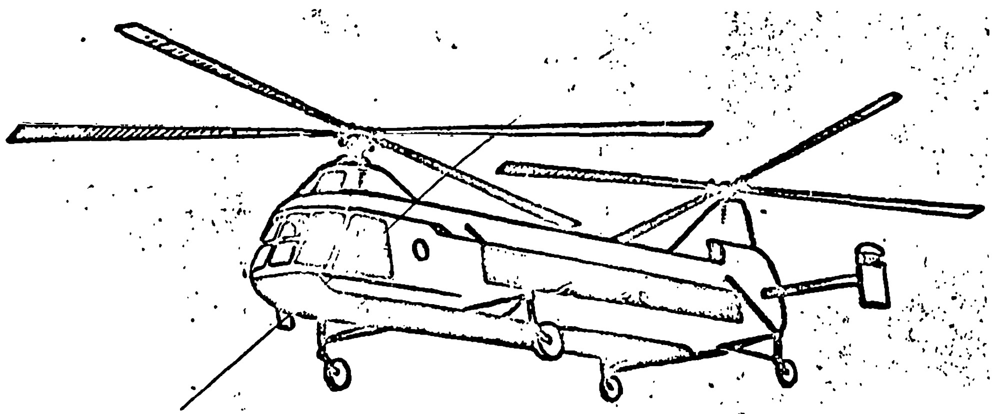

Figure 2. Yak-24 Helicopter.

In 1958 the heavy helicopter Mi-6 was constructed which, up to the present time, has no equal abroad. /5

In 1961 the helicopters Mi-2 and Mi-8 (Figure 3), which have gas turbine engines, were built. At the present time they are in mass production and they will gradually replace the Mi-1 and Mi-4 helicopters.

The ability of a helicopter to fly vertically, and the possibility of motion in every direction, makes the helicopter a very maneuverable flying machine, and since it can operate independent of airfields its boundaries of utilization are considerably widened.

Figure 3. Mi-8 single rotor helicopter.

At the present time helicopters are found in more and more wider application in the national economy. They appear as a basic means of conveyance in locations where it is impossible to utilize ground transport or fixed wing airplanes. Helicopters are utilized in civil construction work and to rescue people and property at times of various natural calamities. Lately helicopters are being widely used in the rural economy. From the examples given, it can be seen that the possibilities of utilizing helicopters as flying machines are far from exhausted.

§ 2. The Helicopter and its Basic Components

Principles of Flight

A helicopter is a heavier than air flying machine that has a lifting force created by a main rotor according to aerodynamic principles.

The basic components of a helicopter are as follows:

Main rotor. Put in motion by the power plant (engine).

Fuselage. Intended for accomodation of crew, passengers, equipment and cargo.

Landing gear, that is, arrangement intended for movement over the ground or for parking. /6

Tail rotor. Provides directional equilibrium and directional control of the helicopter.

Propulsion system which sets in motion the lifting and tail rotors and auxiliary systems.

Transmission transfers the torque from the power plant to the main and tail rotors.

All components of the helicopter are attached to the fuselage or are set in it.

Flight is possible for a flying machine if there is a lifting force counterbalancing its weight. The lifting force of the helicopter originates at the main rotor. By the rotation of the main rotor in the air a thrust force is developed perpendicular to the plane of rotor rotation. If the main rotor rotates in the horizontal plane, then its thrust force T is directed vertically upwards (Figure 4a), that is, vertical flight is possible. The characteristics of the flight depend on the correlation between the thrust force of the main rotor and the weight of the helicopter. If the thrust force equals the weight of the helicopter, then it will remain motionless in the air. If, though, the thrust force is greater than the weight, then the helicopter will pass from being motionless into a vertical climb. If the thrust force is less than the weight, a vertical descent will result.

The plane of rotation of the main rotor with respect to the ground can be inclined in any direction (Figure 4b, c). In this case the rotor will fulfill a two-fold function; its vertical component Y will be the lift force and the horizontal component P — the propulsive force. Under the influence of

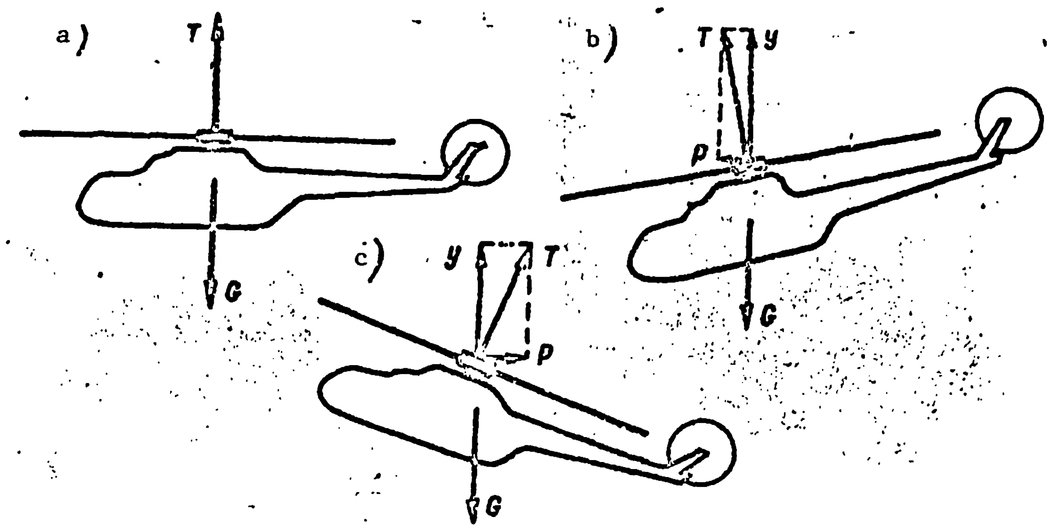

Figure 4. Principle of flight controls of a helicopter.
a - vertical flight; b - horizontal flight forwards;
c - horizontal flight backwards.

this force the helicopter moves forward in flight. /7

If the plane of the main rotor is inclined backwards, the helicopter will move backwards. (Figure 4c). The inclination of the plane of rotation to the right or to the left causes motion of the helicopter in the corresponding direction.

§ 3. Classification of Helicopters

The basic classification of helicopter types is that of the number of main rotors and their disposition. According to the number of main rotors, it is possible to classify helicopters as single rotor, dual rotor and multi-rotor types.

Single rotor helicopters appear in many varieties. Helicopters of the single rotor scheme have a main rotor, mounted on the main fuselage and a tail rotor mounted on the tail structure (see Figure 3). This arrangement, which

was developed by B. N. Yur'yev in 1911, provides a name for one classification.

The basic merit of single rotor helicopters is the simplicity of construction and the control system. The class of single rotor helicopters includes the very light helicopters (flight weight about 500 kgf), and very heavy helicopters (flight weight greater than 40 tons). Some of the deficiencies of the single rotor helicopter are:

Large fuselage length;

A significant loss of power due to the tail rotor drive train (7 - 10% of the full power of the engine);

A limited range of permissible centering;

A higher level of vibration (the long transmission shafts, extending into the tail structure, are additional sources of spring oscillations).

Dual rotor helicopters appear in several arrangements.

Rotors arranged in tandem; this is the most prevalent arrangement (Figure 5a)

Rotors in a transverse arrangement (Figure 5b);

A cross connected rotor scheme (Figure 5c);

A coaxial rotor arrangement (Figure 5d).

The basic merits of helicopters with a tandem rotor arrangement are:

Wider range of permissible centering;

Large fuselage volume; which allows it to contain large-sized loads;

Increased longitudinal stability;

Large weight coefficient.

Helicopters with a tandem arrangement of rotors can have one or two engines, which are located in the forward or aft parts of the fuselage. These helicopters have the following serious deficiencies:

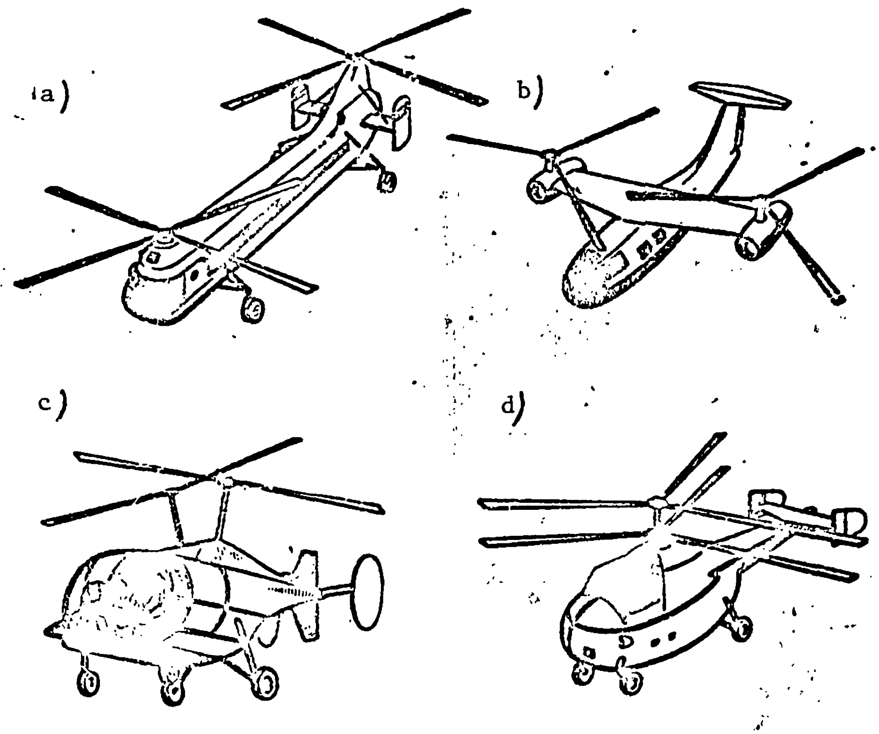

Figure 5. Dual rotor helicopters.

A complicated system of transmission and control; /8

Adverse mutual interaction between the main rotors which causes, in addition, a loss of power;

Complicated landing techniques are required in the autorotation regime of main rotors.

The following advantages are attributed to helicopters with a transverse arrangement of rotors:

Convenient utilization of all parts of the fuselage for crew and passengers, since the engines are located outside the fuselage;

Absence of harmful interaction of one rotor with the other;

Higher lateral stability and controllability of the helicopter;

The presence of an auxiliary wing, where the engines and main rotors are located, allows the helicopter to develop a high speed.

Deficiencies of these helicopters are as follows:

A complicated system of control and transmission;

An increase in size and structure weight due to the presence of the auxiliary wing.

Dual rotor helicopters with cross connected rotors have a considerable advantage over helicopters with transverse rotors; they do not have an auxiliary wing, which reduces the size and structure weight. But, at the same time, with these advantages there is a deficiency, — a complicated transmission /9
and control system.

These helicopters are not produced in the Soviet Union. They are encountered, on occasion, abroad.

The basic advantage of dual rotor helicopters with coaxial rotors is their small size. Their disadvantages:

Complicated structure;

Deficient directional stability;

Danger of collision of the rotor blades;

Considerable vibration.

In the Soviet Union, there are only light helicopters with this rotor arrangement.

Multi-rotor helicopters are not widely used in view of their complex construction.

In all dual-rotor helicopters, the main rotors rotate in opposite directions. In this way the mutual reactive moments are balanced, and the necessity of having a tail rotor is eliminated. Thus the power loss from the engine is reduced.

CHAPTER II

BASIC CHARACTERISTICS OF THE MAIN ROTOR

§ 4. General Characteristics

The main rotor (MR) is a basic component of a helicopter. It is utilized to create the lift and motive force and to control the helicopter.

The basic parts of the main rotor are the hub and the blades.

The blades create the thrust force that is necessary for flight. The hub connects all the blades and serves to fasten the main rotor to the drive shaft. The drive shaft causes the rotor to rotate.

It is possible to subdivide main rotors into three types depending on the structural arrangement:

Those with rigidly fastened blades;
Those with fully articulated blades;
Those with a semi-rigid (flapping) arrangement.

A main rotor with rigidly fastened blades (Figure 6) has the simplest construction and this is its main advantage. But this rotor has inherent and serious disadvantages, which will be discussed in Chapter IV. Therefore, this type of rotor is not utilized in contemporary helicopters. At present, on some light helicopters, as for example the American helicopters, Hughes UH-6A, Hiller EH-1100 and others, main rotors with spring fastened blades are used. /10
These rotors can be considered as a variety of rotor with rigid blades.

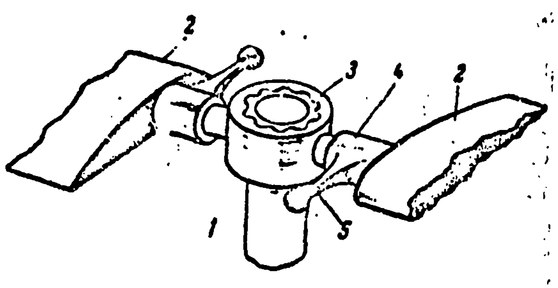

Figure 6. Main rotor with rigidly fastened blades.

1 - Main rotor shaft; 2 - blade; 3 - hub; 4 - axial hinge; 5 - blade balance weight.

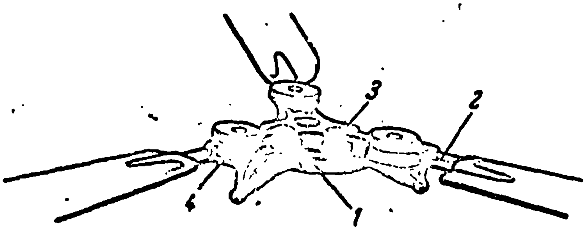

Figure 7. Main rotor with fully articulated blades.

1 - Hub; 2 - axial hinge; 3 - horizontal hinge; 4 - vertical hinge.

The hub of a main rotor with rigid blades has axial hinges, which allow the blades to swing relative to the longitudinal axis as is necessary for control of the rotor.

A main rotor with fully articulated blades appears the most often (Figure 7). Its hub has three hinges for each blade: axial, horizontal and vertical. The main rotor of the semi-rigid arrangement is rarely used. It is not considered in this book.

The hubs of main rotors are made of steel alloy. The blades can be metallic, wooden, or of composite construction. They can also be made of synthetic materials.

§ 5. Geometric Characteristics

A main rotor is characterized by definite geometric parameters: diameter, blade planform shape, blade profile shape, blade incidence angle, the reference area, specific loading and the solidity coefficient.

The diameter of the rotor is the diameter of the circle swept out by the blade tips. It is designated by the letter D and the radius R. The radius of a blade element is designated r (Figure 8a). The ratio of the radius of a blade element to the radius of the rotor is termed the relative radius

$$\bar{r} = \frac{r}{R},$$

which gives $r = \bar{r}R$

The blade planform shape can be rectangular, trapezoidal or a combination (Figure 8b).

In form, the blade resembles the wing of an airplane. The forward edge of the blade is called the leading edge, and the aft edge is called the trailing edge.

Trapezoidal blades have the most uniform distribution of aerodynamic forces along the blade. Rectangular blades are simpler to manufacture, but they have several poor aerodynamic characteristics. The most widely used blades are trapezoidal and rectangular in combination.

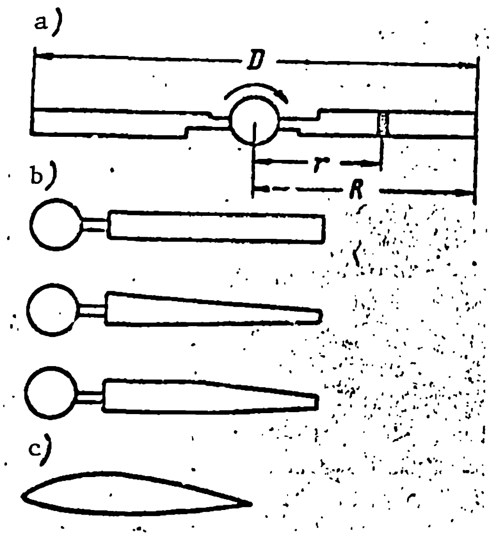

Figure 8. Main rotor parameters.

The profile of the blade is the term used for the form of the blade section perpendicular to the longitudinal axis. The profile of a blade resembles the profile of a wing. Most often double convex asymmetrical sections are used (Figure 8c).

The requirements for a blade profile are:

High aerodynamic efficiency, $K = C_y/C_x$;
Small shift of center of pressure with changes in angle of attack;

The ability to autorotate over a considerable range of angles of attack.

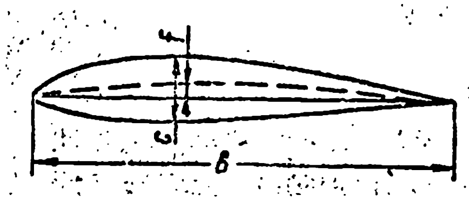

Figure 9. Blade profile parameters.

The profile of the blade is characterized by the relative thickness $\bar{c} = c/b$ and the relative camber $\bar{f} = f/b$ (Figure 9).

According to the relative thickness, the profile is classified as thin ($\bar{c} < 8\%$), medium ($\bar{c} = 8 - 12\%$), or thick ($\bar{c} > 12\%$). Most blades have a relative thickness of $\bar{c} > 12\%$. The use of thick profiles allows an increase in the force resistance of an element and the stiffness of the blade. In addition, the aerodynamic efficiency depends less on the angle of attack for thick profiles. This peculiarity of the profile improves the blade properties in the autorotation regime. Generally, the outermost element of the blade has a greater thickness ratio than at the root.

A relative camber of the blade of $f = 2 - 3\%$ brings the profile form /12
closer to symmetry, which leads to a decrease in the shift of the center of pressure with changes of angle of attack.

The incidence angle of the blade element is termed the angle ϕ; it is formed by the angle between the element chord and the plane of rotation of the main rotor hub (Figure 10). The incidence angle is often called the pitch of the blade element. This is an arbitrary definition. In a more strict definition the pitch of the blade element is the distance H. This distance is obtained from the distance a blade element travels parallel to the chord after one revolution of the main rotor

$$H = 2\pi r \tan \phi$$

Owing to the fact that the pitch of a blade element depends only on the incidence angle ϕ, then in the subsequent discussion we will identify the concept "incidence angle" with the concept "blade element pitch". At different elements of the blade the incidence angles will be different.

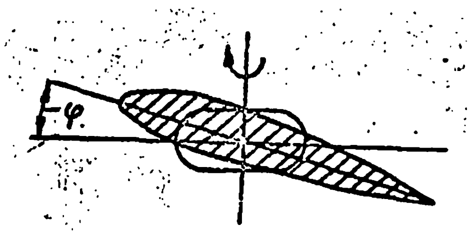

Figure 10. Incidence angle of the blade.

The pitch of the blade is taken as the incidence angle, or the pitch of the blade element, with a relative radius of $\bar{r} = 0.7$. This angle is defined as the incidence angle (pitch) of the main rotor.

As the blade turns relative to the longitudinal axis, the incidence angle changes. Such turning is possible thanks to the presence of the axial hinge. Consequently, the axial hinge of the main rotor blade is intended for pitch alteration.

The alteration of the pitch of the blade elements over the radius of the main rotor is termed the geometric twist of the blade.

At the root of the blade elements, the incidence angles are the largest, while at the tip they are the smallest (Figure 11). Geometric twist improves the operating conditions of the blade elements, and the angles of attack approach the optimum. This causes an increase of the thrust force of the lifting rotor of 5 - 7%. Therefore, geometric twist increases the useful loading of the helicopter at constant engine power.

Owing to geometric twist a more uniform force loading on the blade element is achieved and the speed, at which flow breakdown occurs on the retreating blade, is increased. The majority of blades have a geometric twist which does not exceed 5 - 7°

Stiffness is understood to mean the ability of the blade to retain its form. With great stiffness, even force loading is not capable of deforming the structure and external shape of the blade. With small stiffness the blade becomes flexible and easily yields to deformation, that is, the blade is strongly bent and twisted. If the flexibility is too great, the optimum /13

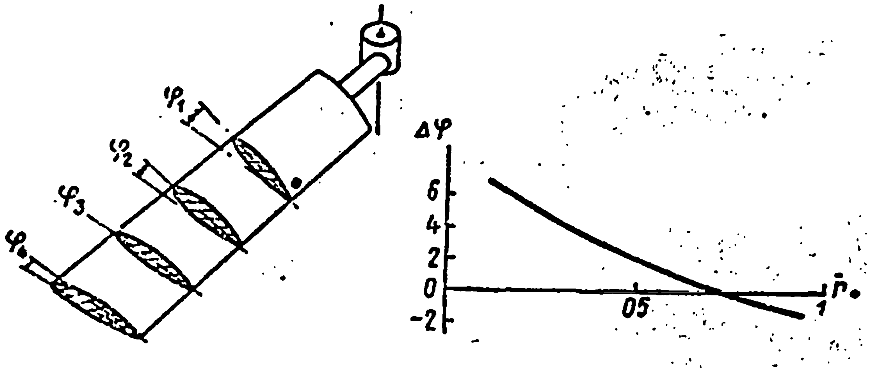

Figure 11. Geometric twist of the blade.

twist cannot be maintained on the blade. This leads to inferior aerodynamic characteristics of the main rotor.

In order to obtain great stiffness, it is necessary to increase the size of the load supporting elements, which leads to increased weight of the blade. Unnecessarily high stiffness leads to an increase of vibration of the main rotor.

The greatest stiffness is obtained with blades of metal or of continuous wooden construction, but the latter are very heavy and are utilized only on light helicopters.

The area swept out by the main rotor is the area of the circle described by the blade tips

$$F = \pi R^2 = \pi \frac{D^2}{4}.$$

This characteristic of the main rotor has approximately the same importance as the wing area of a fixed wing airplane, that is, it is similar to the lifting surface area.

The disk loading, based on the swept area, is defined as the ratio of helicopter weight to area, that is, the area swept out by the main rotor.

$$P = \frac{G}{F},$$

where, P = specific loading, kgf/m^2;

G = helicopter weight, kgf;

F = swept area, m^2.

Contemporary helicopters have specific loadings that vary from 12 to 25 $kgf.m^2$ (or 120 - 150 N/m^2).

The solidity coefficient is equal to the ratio of the total planfrom area of all the blades to the area swept out by the main rotor.

$$\sigma = \frac{S_B k}{F}$$

where, S_B = planform area of one blade, m^2;

k = number of blades

Contemporary main rotors have perhaps from 2 to 6 blades. Most often there are 3 - 4 blades on light helicopters and 5 - 6 blades on heavy helicopters. The space factor has a value from .04 to .07. This means that /14 4 - 7% of the area swept out by the rotor is taken up by the blades. The larger the space factor, within the limits indicated, the larger the thrust developed by the rotor. But if the space factor exceeds .07, then the forces of resistance to rotation are increased and the blade efficiency of the main rotor is decreased.

§ 6. Basic Regimes of Operation

The operating conditions of the main rotor or its working regime are defined as the position of the main rotor relative to the air stream. Depending

on the position, two basic working regimes are considered, those of axial and translational flow.

The axial flow regime is the term used for the operating condition of the main rotor where the axis of the hub is parallel to the oncoming free stream flow. In the axial flow regime the free stream passes perpendicular to the plane of rotation of the main rotor hub (Figure 12a). This regime covers the hovering, vertical climb and vertical descent conditions of the helicopter main rotor. An important feature of the axial flow regime is that the location of the blade of the rotor, relative to the oncoming free stream, is not changed. Consequently, the aerodynamic forces on the blade as it moves around the circle are not changed.

The oblique flow regime is the term used for the operating conditions of the main rotor, where the airstream approaches the rotor in a direction not parallel to the axis of the hub. An important difference of this regime is that, as the blade moves around in a circle, it continuously changes its location relative to the flow approaching the rotor. As a consequence, the velocity of the flow at each element is changed and also the aerodynamic forces on the blade. The translational flow regime occurs in the horizontal flight of a helicopter and in flight inclined upwards and downwards.

From consideration of the operating conditions, one can see that the position of the main rotor in the airflow is important. This position is determined by the angle of attack of the main rotor.

The angle of attack of the main rotor is termed angle A, and it is formed by the plane of rotation of the hub and the flight velocity vector, or by the undisturbed flow approaching the rotor. The angle of attack is positive if the flow approaches the rotor from below (Figure 12b). If the flow approaches the rotor from above, the angle of attack is negative (Figure 12c). If the airflow approaches the rotor parallel to the plane of rotation of the hub, the angle of attack is zero (Figure 12d).

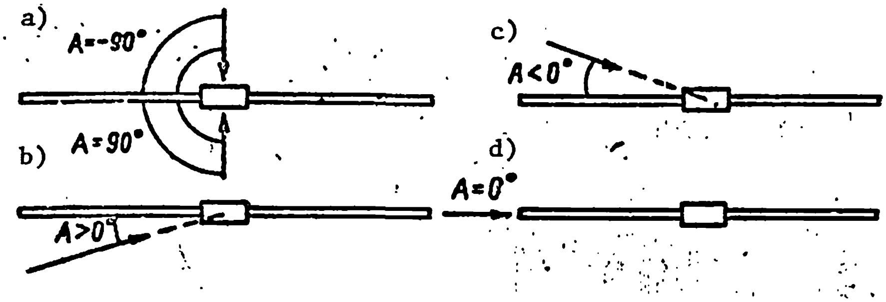

Figure 12. Operating regimes and angle of attack of the main rotor.

It is not difficult to observe the connection between the operating regime of the main rotor and the angle of attack:

In the axial flow regime, the angle of attack of the main rotor A = ± 90°.

In the oblique flow regime, A ≠ ± 90°. /15

If the angle of attack A = 0°, the operating regime of the main rotor is termed the planar flow regime.

§ 7. The Operating Regime Coefficient of the Main Rotor

A special quantity is introduced to characterize the operating regimes of the main rotor — the operating regime coefficient(*)

The operating regime coefficient of a main rotor, μ, is defined as the ratio of the projection of the flight velocity vector on the plane of rotation of the hub to the circular velocity of the blade tip. The projection of the flight velocity vector, or the undisturbed flow, on the plane of rotation of the hub is equal to the product of V cos A (Figure 13).

(*)Translator's note: This is the tip speed ratio.

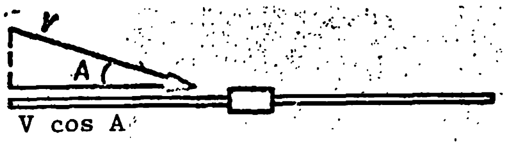

Figure 13. Projection of the flight velocity on the hub plane of rotation.

Then we have

$$\mu = \frac{V \cos A}{\omega R}.$$

In the axial flow regime, when $V = 0$, or $\cos A = 0 (A = 90°)$, $\mu = 0$.

Consequently, the equation $\mu = 0$ indicates the axial flow regime. If $\mu > 0$, this is the index of the transverse flow regime. The larger the coefficient μ, the larger the effect of transverse flow. The coefficient μ for contemporary helicopters varies from 0 to 0.4. In most cases the angle of attack of the main rotor does not exceed 10°. Since $\cos 10° \approx 1$, then it is possible to define μ by the approximate formula

$$\mu = \frac{V}{\omega R}.$$

CHAPTER III

OPERATION OF THE MAIN ROTOR IN THE AXIAL FLOW REGIME /16

During the rotation of the main rotor, a thrust force arises that creates lift and motion forces for the helicopter. The question of the origin of this thrust force is a basic question in the study of the main rotor operation. Several theories exist that explain the origin of the thrust force. We consider the physical aspects of two such theories.

§ 8. Impulsive Theory of an Ideal Rotor

In this theory an ideal rotor is considered — *that is, a rotor that* operates without losses. Such a rotor receives its energy from the engine, and all of it is transformed into work by displacement of the air mass along the axis of rotation.

If the rotation of the rotor in the hovering regime is considered — that is, when there is no translational motion of the helicopter and its speed is zero — the air is attracted by the rotor from above and from the sides (rotor induced flow) and it is deflected downwards (Figure 14). A flow of air is established through the area swept out by the rotor. The parameters of this flow are characterized by the inflow velocity V_i (the speed of the flow in the plane of rotation and the main rotor), by the downwash velocity of the flow of V_D (the speed of the flow at a certain distance from the plane of rotation of the main rotor), by the increase of pressure in the flow ΔP, and by the change of speed along the axis of rotation.

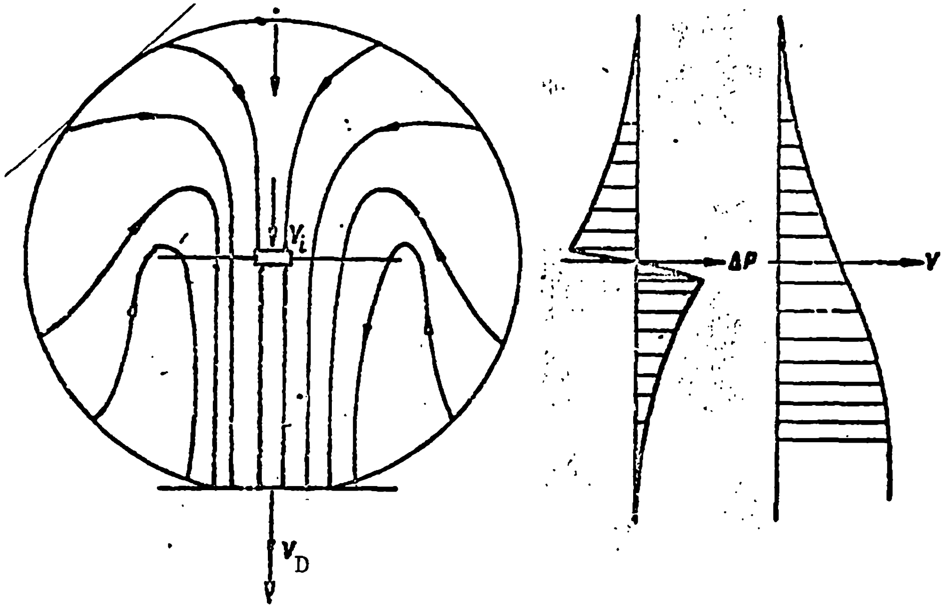

Figure 14. Operation of an ideal rotor according to impulsive theory.

By deflecting the air downwards with a force T, the rotor receives a force from the air in the upward direction (action equals reaction). This force will be the thrust force of the rotor. But from mechanics, it is known that a force equals the product of the mass of a body times the acceleration that the body receives under the action of the force. Therefore,

$$T = m_s a, \tag{1}$$

where m_s = mass of air per second, flowing through the area swept out by the rotor;

a = acceleration in the flow.

The mass of air per second is determined from the formula:

$$m_s = \rho F V_i, \tag{2}$$

where ρ = air density;

F = area swept out by the rotor;

V_i = induced flow velocity (inflow velocity)

As is known, the acceleration is equal to the change of velocity in unit time,

$$a = \frac{dV}{dt},$$

where dV = increase of flow velocity

dt = time

If we take $dt = 1$ second, the acceleration a is numerically equal to the velocity increase, that is, $a = dV$.

Let us clarify the value of dV. Considering the operation of the rotor in the hovering regime, it is not difficult to see (Figure 14) that the air at a certain distance from the rotor is stationary — that is, its velocity relative to the rotor is zero.

Beneath the rotor, the air moves at the inflow velocity, which means that the velocity increase $dV = V_D$. Then utilizing Formulas (1) and (2) we obtain

$$T = m_s a = m_s V_D = \rho F V_i V_D. \tag{3}$$

In order to arrive at a final conclusion, it is necessary to ascertain the relationship between the inflow velocity V_i and the downwash velocity V_D. We use the law of conservation of momentum: 'The impulse of a force equals the increase of momentum'.

It will be recalled that the impulse of a force is the product of force and time. If the time $dt = 1$ second, then the impulse of the force numerically equals the force.

The product of the mass of a body and the velocity increase is termed the increase of momentum: $m_s dV = m_s V_D$. This means that, based on the fundamental law of conservation of momentum, we obtain the thrust force; /18

$$T = m_s V_D. \tag{4}$$

The work per second by the main rotor with respect to the downwash will have the value;

$$N = TV_l. \tag{5}$$

But because we are considering an ideal rotor — that is, a rotor without losses — then, as a consequence, all of the work is changed into kinetic energy of the flow leaving the rotor. The kinetic energy is determined by the formula

$$E_k = \frac{m_s V_D^2}{2}.$$

Utilizing Formula (4), we find;

$$E_k = \frac{T}{2} V_D. \tag{6}$$

Equating Formulas (5) and (6) on the basis of the theory of an ideal rotor, we obtain

$$TV_i = \frac{T}{2} V_D \qquad \text{or} \qquad 2V_i = V_D.$$

Thus Formula (4) finally takes the form

$$T = {}_{\rho}FV_i V_D = {}_{\rho}FV_i 2V_i \quad \text{or} \quad T = 2_{\rho}FV_i^2. \tag{7}$$

The conclusion is that the thrust force, developed by the main rotor, is proportional to the air density, the area swept out by the rotor and induced velocity squared.

In order to determine on what the induced velocity depends, it is necessary to consider another theory that explains the origin of the thrust force of the main rotor.

§ 9. Blade Element Theory

In accordance with this theory, each element of the blade is considered as a small wing, which moves in a circular trajectory with speed $u = \omega r$ (Figure 15a). If the profile of the blade were symmetrical and the incidence angle $\phi = 0$, there would be no deflection of air downwards, and V_i and T would be equivalent to zero.

For an asymmetrical profile and $\phi > 0$, the airflow approaching the blade element is deflected downwards. This deflection, and as a consequence, the induced velocity will be larger, the larger the incidence angle of the element, and the greater the angular velocity or the rotation of the main rotor (Figure 15b).

Adding the vectors of circular and induced velocity, we obtain the resulting vector $\bar{w} = \bar{u} + \bar{V}_i$ /19

The angle α between the chord of the blade element and the resultant velocity vector is termed the angle of attack of the blade element. The aerodynamic forces, arising from the main rotor blade, depend on this angle.

Examining the spectrum of the streamlines around a blade element, it can be observed that the streamlines have the same form as the spectrum of a wing (Figure 15c). On this basis we can state that the air pressure on the blade upper surface will be less than on the lower surface. Owing to the

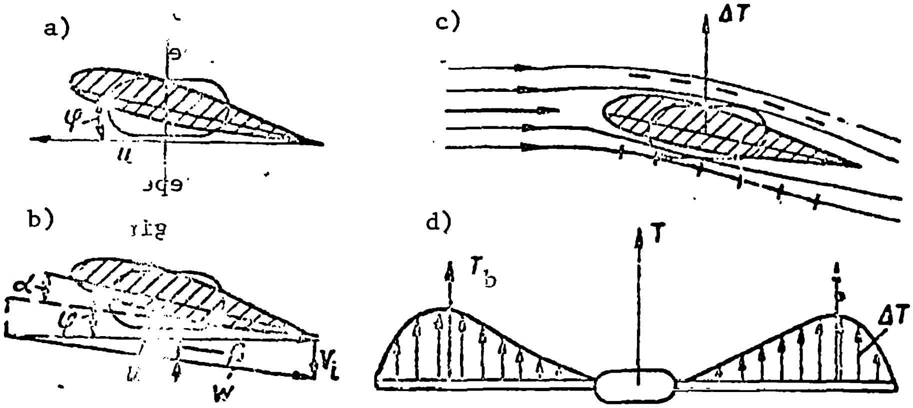

Figure 15. Development of thrust force according to blade element theory

difference in pressure, there arises an element of thrust force ΔT (Figure 15c). If all the elements of force are summed, we obtain the thrust force of the entire rotor

$$T = T_B k$$

where k = number of blades

T_B = thrust of blade; $T_B = \Sigma \Delta T$

To determine the force of the main rotor, it is possible to utilize the formula for the lift force of a wing:

$$T = C_T F \frac{\rho}{2} (\omega R)^2, \qquad (8)$$

where C_T = thrust coefficient.

Because $\omega R = u$ (u is the circular velocity of the blade tip), it is possible to write the formula for the thrust force in the following form:

$$T = C_T F \frac{\rho}{2} u^2. \qquad (9)$$

The conclusion is that the thrust force of the main rotor is proportional to the thrust coefficient, the area swept out by the rotor, air density, and the square of the circular velocity of the blade tip.

For a given rotor at a constant air density, the thrust depends on the number of revolutions and the thrust coefficient. The thrust coefficient depends on the pitch of the rotor (Figure 16). /20

The conclusions that have been outlined according to "impulsive theory" and "blade element theory" do not contradict each other, but are mutually supplementary. On the basis of these conclusions, it is possible to state that, in order to increase the thrust force of the main rotor, it is necessary to increase the pitch or the revolutions, or both of them at the same time. Besides the thrust force, the rotation of the rotor gives rise to forces that resist rotation. We will consider these forces in the next section.

§ 10. Forces Resisting Rotation of the Main Rotor

The forces resisting rotation are called the aerodynamic forces operating in the plane of rotation of the hub and directed against the rotation.

At each blade element, its own element of force arises to resist rotation. In a similar way to the drag force of a wing, the elements of the forces resisting rotation consist of the forces of profile and induced drag.

Profile rotational resistance ΔQ_p is an aerodynamic force that arises because of the difference of air pressure in the forward and aft parts of the blade, and also due to the friction of the air in the boundary layer. In general, the profile drag depends on the number of revolutions of the main

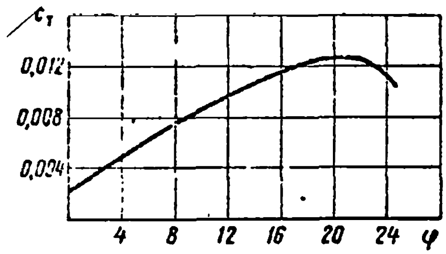

Figure 16. Relationship between the thrust coefficient and the pitch of the main rotor.

rotor, the condition of the blade surface and the form of the profile. It is changed very little by changes in the pitch of the rotor (Figure 17a).

Induced resistance arises owing to the induced cross flow on the blade of the main rotor. The induced cross flow deflects the vector of elementary thrust force by an angle β backwards relative to the axis of the hub (Figure 17b). If the vector of the deflected elementary thrust force ΔT is projected on the rotation plane of the hub, we obtain the vector of an elementary induced force that resists rotation ΔQ_i.

The induced rotational resistance depends, principally, on the pitch of the main rotor (with an increase in pitch, it increases). Profile and induced drag, just like the thrust force, depend on air density.

The reactive moment of the main rotor. The elementary rotational resistance forces arise on each element of the blade. Combining the elementary forces of one blade, we obtain their resultant $Q_b = \Sigma\Delta Q$ (Figure 17c). /21

Since the forces resisting rotation are directed opposite the rotor rotation, their geometric sum (resultant) is zero and does not lead to translational motion of the main rotor. But the forces resisting rotation create a torque about the hub axis, termed reactive, and sometimes termed the rotational resistance torque M_r (see Figure 17c)

$$M_r = Q_b r_Q k,$$

where r_Q is the radius of the blade center of pressure;
k is the number of blades.

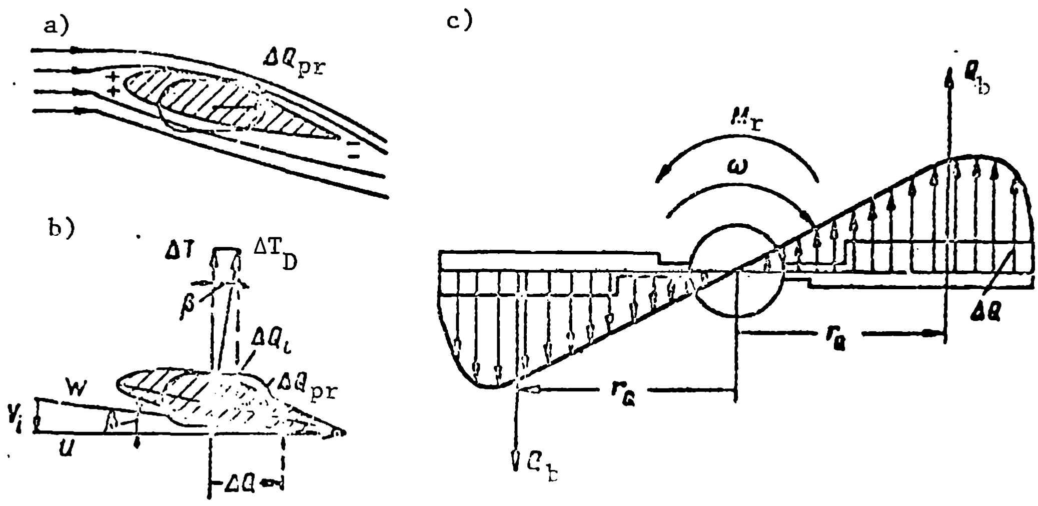

Figure 17. Main rotor rotational resistance forces.

The reactive torque depends on those same factors which determine the magnitude of the forces resisting rotation, i.e., rotor pitch, rotor rpm, blade surface condition and shape, and air density.

The reactive torque is directed opposite the rotor rotation, consequently this torque is a retarding torque; it tends to stop the rotor and reduces its angular velocity of rotation.

§ 11. Power and Torque Required to Rotate Main Rotor

In order for the main rotor to turn, the action of the reactive torque must be overcome, i.e., driving torque must be supplied to the rotor.

The torque M_{tor} which must be supplied to the main rotor is termed the /22 required torque. In magnitude, it equals the reactive torque — in direction, it opposes the latter

$$M_{tor} = m_{tor} F \frac{\rho}{2} (\omega R)^2 R \qquad (10)$$

where m_{tor} is the torque coefficient.

The torque coefficient is a composite quantity, i.e.,

$$m_{tor} = m_{tor_{pr}} + m_{tor_i}$$

where $m_{tor_{pr}}$ is the part of the torque coefficient due to profile drag forces. This part depends on the condition of the blade surface, the rotor rpm, and the blade shape;

m_{tor_i} is the part of the torque coefficient due to the induced drag forces and depends primarily on the main rotor pitch (Figure 18).

The formula for the required torque, and also the curve of this torque coefficient versus rotor pitch, makes it possible to conclude that the main rotor required torque will increase with increase of the pitch, rpm, and air density.

We recall that power is work per unit time. The concept of the power required to turn the main rotor can be obtained if we examine the work expended in overcoming the forces resisting the rotation of a single blade, and then the work expended in overcoming the reactive torque of the entire rotor (Figure 19).

The work of a single blade during one revolution of the main rotor is

$$A_b = Q_b 2\pi r_Q .$$

The main rotor work per second, i.e., the power required, is /23

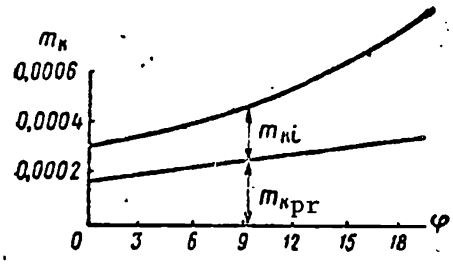

Figure 18. Torque coefficient versus main rotor pitch.

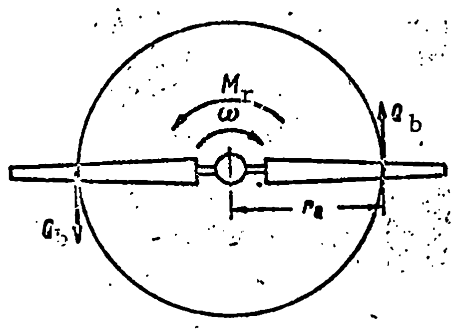

Figure 19. Action of rotational resistance forces.

$$N_{req} = A_b k n_s$$

where k is the number of blades;

n_s is the rps.

Consequently,

$$N_{req} = Q_b 2\pi r_Q k n_s$$

since

$$Q_b r_Q k = M_p \text{ and } 2\pi n_s = \omega,$$

then

$$N_{req} = M_p \omega . \qquad (11)$$

Both the power required and the torque required for the main rotor change with change of the pitch, rpm, and air density. In order to turn the

rotor, engine power equal to the power required must be supplied to the rotor shaft. This equality is the condition for constant rpm

$$N_{sup} = N_{req}$$

where N_{sup} is the power supplied to the rotor from the engine.

If the power supplied $N_{sup} > N_{req}$, the rotor rpm will increase. However, if $N_{sup} < N_{req}$ the rotor rpm will decrease.

§ 12. Main Rotor RPM Control

The main rotor rpm will change both with change of the power supplied, i.e., the engine power, and with change of the power required, i.e., with change of the main rotor reactive torque. The magnitude of the thrust developed by the main rotor changes with change of the rpm.

We need to know the optimal rotor rpm, i.e., is it better to increase the thrust by increasing rpm or pitch? Moreover, we need to know how to maintain the optimal main rotor rpm with variation of the magnitude of the thrust.

The answer to the first of these questions can be obtained by examining the characteristic termed specific thrust. Main rotor specific thrust is a quantity equal to the ratio of the thrust developed by the rotor to the power required to turn the rotor

$$q = \frac{T}{N_{req}}. \qquad (12)$$

The specific thrust shows the number of units of thrust per unit of power expended by the engine in turning the rotor. The larger the specific thrust, the more efficient the main rotor. /24

We substitute the values of the thrust from (8) and the power required from (10) and (11) into (12).

Then

$$q = \frac{c_T F \frac{\rho}{2} \omega^2 R^2}{M_{tor} \omega} = \frac{c_T F \frac{\rho}{2} \omega^2 R^2}{m_{tor} F \frac{\rho}{2} \omega^3 R^3} .$$

After simplifying, we obtain

$$q = \frac{c_T}{m_{tor} \omega R}, \text{ if } \frac{c_T}{m_{tor}} = \text{const and } \omega R = u$$

then we finally obtain

$$q = \frac{\text{const}}{u} .$$

Consequently, to increase the thrust we should reduce the main rotor tip speed. This means that it is better to increase the thrust by increasing the main rotor pitch at minimal rpm. Here, it must be emphasized that there is a minimal permissible rpm for every rotor. Reduction of the rpm below the minimal acceptable value leads to flight safety problems, deterioration of helicopter controllability and stability.

This conclusion is very important, as it provides an answer to the question of why heavy and complex main rotor reduction gearboxes are installed in helicopters. These reducers make it possible to connect the main rotor shaft, which rotates at a low angular velocity, with the engine shaft, which rotates with an angular velocity 10-15 times that of the rotor.

Thus, we have established that it is advisable to turn the main rotor at low speed and increase thrust by increasing the pitch. But increase of the pitch leads to increase of the reactive torque and, therefore, increase of the power required. This means that in order to maintain constant rotor rpm

the power supplied to the rotor must be changed at the same time the pitch is changed. The main rotor and the engine must be controlled simultaneously. Simultaneous control is accomplished with the aid of a special lever, termed the "collective-throttle" lever. This lever is installed in an inclined position to the left of the pilot's seat. If the collective-throttle lever is displaced upward, both the main rotor pitch and the engine power are increased simultaneously, and the main rotor rpm remains approximately constant. The throttle twist grip is located on the end of this lever. The engine power alone, and therefore the main rotor rpm, can be altered by rotating this grip.

§ 13. Techniques for Counteracting Main Rotor Reactive Torque /25

The reactive torque retards rotation of the main rotor and causes the helicopter to turn in the direction opposite that of the rotor. The turning action of the reactive torque is counteracted in various ways. On single-rotor helicopters the reactive torque is balanced by the tail rotor thrust moment (Figure 20).

Since the helicopter turns about its center of gravity, the tail rotor thrust moment is defined relative to the vertical axis of the helicopter. The helicopter will not turn about the vertical axis if the reactive torque equals the tail rotor thrust moment, which is defined by the formula

$$M_{t.r} = T_{t.r} l$$

where l is the distance from the helicopter center of gravity to the tail rotor.

From the formula $N_{req} = M_p \omega$ we can determine the magnitude of the main rotor reactive torque and the equal tail rotor thrust moment

$$M_r = M_{t.r} = \frac{N_{req}}{\omega}.$$

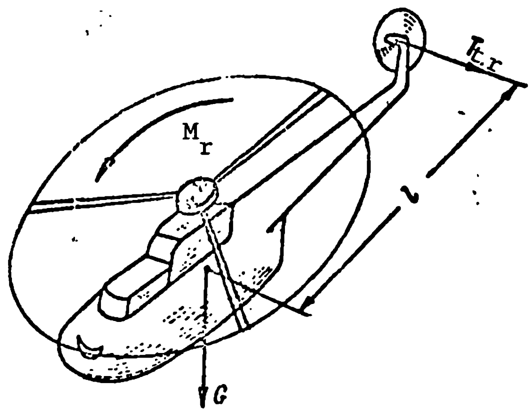

Figure 20. Balancing of main rotor reactive moment on a single-rotor helicopter.

Knowing the distance l, we find the tail rotor thrust

$$T_{t.r} = \frac{M_{t.r.}}{l} .$$

Now it is not difficult to explain the purpose of the helicopter tail rotor. The tail rotor of the single-rotor helicopter is intended to create a thrust whose moment balances the main rotor reactive torque and thereby prevents rotation of the helicopter around the vertical axis. Directional control of the helicopter is accomplished by varying the tail rotor thrust and its moment about the helicopter vertical axis.

In helicopters with two main rotors, the turning action of the reactive torques is automatically eliminated — the main rotors turn in opposite directions and their reactive torques balance one another.

§ 14. Techniques for Transmitting Torque from Engine to Main Rotor

/26

With regard to the technique used to create and transmit torque, modern helicopters can be divided into two groups:

1) those with reactive drive;
2) those with mechanical drive.

In helicopters with reactive drive the engines are located at the tips of the main rotor blades (Figure 21a). In this case, the torque can be expressed as the product of the reactive engine thrust P_{eng} by the main rotor radius R and the number k of blades

$$M_{tor} = P_{eng} Rk.$$

The torque balances directly the moment resisting rotation; therefore, the helicopter will not turn.

Characteristic for the helicopter with reactive drive are simplicity of its construction and low weight. It has no power expenditure to rotate a tail rotor, less vibration, and there is the possibility of obtaining high main rotor thrust with low thrust of the jet engine located at the tip of the blade.

Any type of reactive engine can be used as the reactive engine at the tip of the blade. However, at the present time the so-called compressor drive is most often used, i.e., reaction nozzles are located at the tips of the blades and are supplied with compressed air from a gas turbine engine or a special compressor.

The reaction-driven helicopter is still in the experimental stage. This is a result of difficult technical problems, the primary ones being:

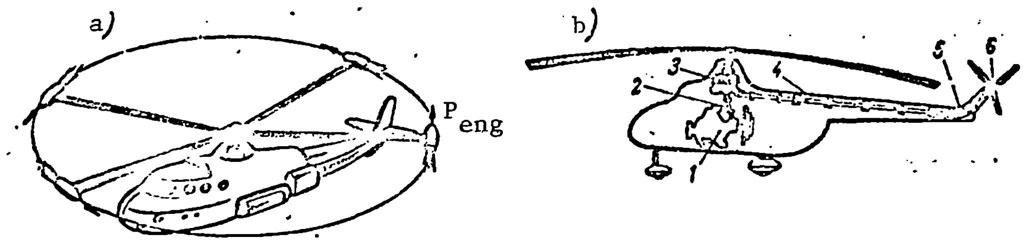

Figure 21. Techniques for transmitting power to main rotor:

a) reactive drive; b) mechanical drive;

1) engine gearbox; 2) main transmission shaft; 3) main rotor gearbox; 4) tail rotor driveshaft; 5) intermediate gearbox; 6) tail rotor gearbox.

high fuel consumption and low efficiency (2-3%) of the reaction drive;

complexity of the construction of the hub and blades, in which plumbing /27 must be provided;

complexity of the design of a reaction engine which will operate reliably when subjected to the high centrifugal force and the varying airstream direction;

deterioration of the aerodynamic characteristics of the main rotor owing to the engines located on the blades.

Helicopters with mechanical drive are those in which the torque transmitted from the engine to the main and tail rotors by means of a special assembly, termed a transmission (Figure 21b).

The transmission includes the following basic units:

Reducers;
Shafts;
Shaft supports and connections;
Clutches;
Reducer mount frames;
Main rotor brake.

The transmission reducers include:

Main rotor reducers;

Intermediate reducers;

Airframe-mounted engine reducers;

Tail rotor reducers.

The main rotor reducer is provided to reduce the rotor shaft speed. The need for this reduction was explained above. Characteristic of this reducer is the high reduction ratio — from 1:8 to 1:14. Two-stage simple reducers are used on light helicopters; usually two-stage planetary reducers are used on the intermediate and heavy helicopters. The torque to the tail rotor is transmitted through the main rotor reducer. When the main rotor turns, the tail rotor is also automatically rotated. Thus, the main and tail rotors always constitute a single system and cannot rotate separately.

The intermediate reducers are installed in order to change the transmission direction (for example, at the juncture of the tail boom and the aft vertical fin). These reducers do not change the rpm, and consist of two conical gears.

The airframe-mounted engine reducers are used to transmit the torque from the horizontal engine shaft to the vertical transmission shaft. They are located in the engine case, and are used when the engine shaft axis is horizontal.

The tail rotor reducers are provided to transmit torque to the tail rotor /28
shaft and to reduce tail rotor shaft rpm. The mechanism for controlling the tail rotor is located in its reducer.

The torque is transmitted by the transmission shafts. The transmission of a single-rotor helicopter includes:

Main transmission shaft;

Tail rotor driveshaft.

The main transmission shaft transmits the torque from the engine to the main rotor reducer.

As a rule, the tail rotor driveshaft consists of several sections and transmits the torque from the main rotor reducer to the tail rotor reducer and its length is 8-10 m. This shaft is a source of additional vibration of the helicopter.

All the transmission shafts rotate at high angular speed. Increase of the angular speed reduces the loading on the shaft for transmission of the same power. If

$$N_{req} = M_{tor}\omega, \text{ then } M_{tor} = \frac{N_{rec}}{\omega}.$$

The shaft supports prevent deflection and bending vibrations (whipping) of the long shafts. Ball bearings with elastic spacers are used as the supports. The shafts are connected with one another and with the other parts of the transmission by means of universals and flexible couplings; in addition to the interconnecting couplings there are starting, engaging, and freewheeling clutches.

On some helicopters all three of these clutches are combined into a single unit, located in the engine case together with the reducer. The free-wheeling clutch is most frequently made in the form of a separate unit. The starting clutch is a unit of the friction type and is intended for smooth connection of the transmission shaft with the engine shaft. When this type of connection is used, there is slippage of one shaft relative to the other until the speeds of the driving and driven shafts become the same. This clutch transmits the small torque from the engine to the transmission when the engine is operating at low speed. The starting clutch provides smooth rotation of the main and tail rotors without jerking. When the transmission is engaged, the main clutch (most often of the dog type) is activated and connects the engine and transmission shafts rigidly together. The total torque is transmitted from the engine to the main and tail rotors through this

clutch. The freewheeling clutch is designed to transmit torque in one direction only — in the direction of rotation of the rotor. It provides automatic disconnect of the engine from the transmission if there is a reduction of the engine rpm. This is necessary in the main rotor autorotation regime if there is an engine failure in flight. Moreover, the presence of the freewheeling clutch leads to reduction of the inertial loads on the main rotor shaft when there is a change of engine operation. As a rule, the freewheeling clutch is located in the main rotor reducer case, between the main transmission shaft /29 and the reducer shaft. The main rotor brake is designed for rapid deceleration of the transmission after shutting down the engine on the ground.

The helicopter transmission is quite heavy, and therefore reduction of the weight of its individual components is of primary importance.

§ 15. Main Rotor Power Available

The power required to turn the main rotor is supplied to the rotor from the engine through the transmission. But the rotor does not receive all the power the engine develops, since part of this power is expended for other purposes and does not reach the rotor. The overall power losses are made up of the losses in:

Turning the tail rotor;
Turning the engine cooling fan;
Overcoming friction in the transmission components;
Driving the accessories;
Overcoming air drag on fuselage and other parts of the helicopter.

Let us examine the magnitudes of these losses, or the energy balance of the helicopter.

On the average, 8% of the engine power is expended in turning the tail rotor ($N_{t.r}$);

The fan absorbs 5% (N_{fan});

The friction in transmission absorbs 7% (N_{trans});

The accessories absorb 1% (N_{acc});

Helicopter parasite drag absorbs 2% (N_{par}).

That portion of the engine power which is supplied to the main rotor is called the power available. It is defined as the difference between the effective engine power and the sum of the losses

$$N_{avail} = N_e - (N_{t.r} + N_{fan} + N_{trans} + N_{acc} + N_{par}).$$

The ratio of the power available to the effective engine power is termed the power utilization coefficient

$$\zeta = \frac{N_{avail}}{N_e}$$

hence

$$N_{avail} = N_e \zeta .$$

The difference $1 - \zeta = \zeta_l$ is called the power loss coefficient.

For single-rotor helicopters, the average power utilization coefficient is 0.75-0.80, and the average power loss coefficient is 0.25-0.20. The power utilization coefficient, and consequently the power available, vary with variation of the helicopter flight speed. The speed dependence of the power available is shown in Figure 22. /30

The following conclusions can be drawn from this figure:

1) The effective engine power is independent of the flight speed;
2) The overall power loss decreases with speed up to 80-100 km/hr and then increases with further increase of the flight speed;
3) The power available increases with increase of the flight speed to 80-100 km/hr and then decreases;

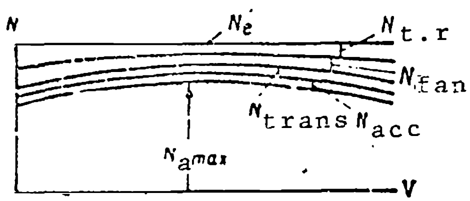

Figure 22. Main rotor power available versus speed.

4) The maximal power available is obtained at a flight speed from 80 to 100 km/hr for most helicopters.

§ 16. Main Rotor Thrust in Vertical Climb and Vertical Descent

Main rotor thrust in vertical climb. It was established above that the thrust of the ideal main rotor in the hovering regime is defined by the formulas

$$T = m_S V_{dw} \quad \text{or} \quad T = 2\rho F V_i^2.$$

The first formula is of a general nature and is applicable for all axial-flow regime cases. The second is applicable only for determining the thrust in the hovering regime.

During vertical climb, the magnitude of the air mass flowrate m_S through the swept area changes. This is seen from the schematic of main rotor motion during vertical climb (Figure 23a). The rotor travels upward with the velocity V. We can say that an undisturbed flow caused by this motion approaches the rotor (principle of reversibility of motion). In the plane of rotor rotation, the flow velocity V_1 will be /31

$$V_1 = V + V_i.$$

If the air mass flowrate is defined as $m_S = \rho F V_1$, then the thrust is

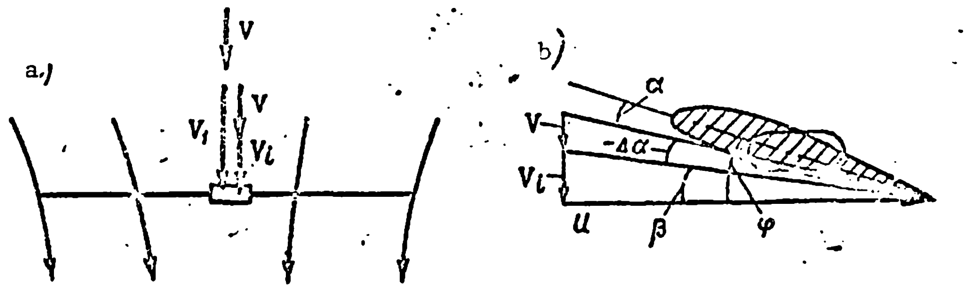

Figure 23. Operation of main rotor in vertical climb.

defined as $T = m_S V_{dw}$ or $T = \rho F V_1 V_{dw}$, and since the equality $V_{dw} = 2V_i$ is also valid for vertical climb, the thrust will be

$$T = \rho F V_1 V_{dw} = 2\rho F V_1 V_i.$$

Comparing the main rotor thrust $T = 2\rho F V_i 2$ in the hovering regime, and the thrust $T = 2\rho F V_1 V_i$ in the vertical climb regime, we can say that the thrust in the climbing regime is higher than that in the hovering regime, since $V_1 > V_i$. But this conclusion would be valid only if the induced velocity V_i did not change with change of the rotor motion velocity. In actuality, the induced velocity decreases with increase of the translational velocity, which leads to reduction of the main rotor thrust.

This means that the main rotor must develop more thrust during vertical climb than the weight of the helicopter. The dependence of the main rotor thrust on speed can also be explained from the viewpoint of blade element theory. In helicopter hovering, the blade element angle of attack depends on the pitch and the induced flow velocity (Figure 15b).

With increase of the climb velocity, the angle of attack of the main rotor blade element decreases, and therefore the main rotor thrust coefficient

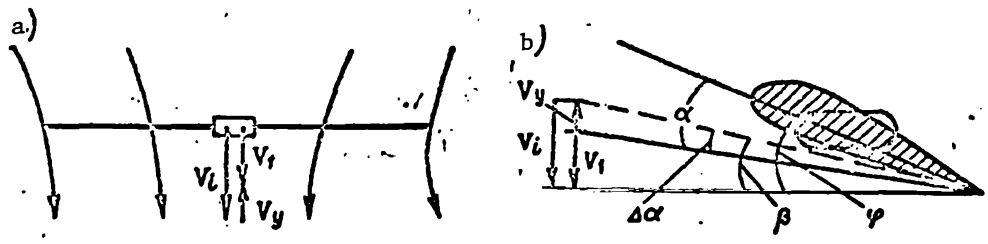

Figure 24. Operation of main rotor in vertical descent.

decreases, which in turn leads to reduction of the main rotor thrust, since $T = C_T F \frac{\rho}{2} (\omega R)^2$ (Figure 23b).

Main rotor thrust in vertical descent. During vertical descent (Figure 24a), the undisturbed flow approaches the main rotor from below with the velocity V_y; therefore, the flow velocity in the plane of rotation of the main rotor is $V_1 = V_i - V_y$, i.e., it will be less than during hovering.

Main rotor thrust in vertical descent is defined by the same formula as for vertical climb $T = C_T F \frac{\rho}{2} u^2$ or $T = 2\rho F V_1 V_i$.

The main rotor blade element angle of attack is increased during vertical /32
descent by the amount $\Delta\alpha$ as a result of the vertical descent velocity, which leads to increase of the coefficient C_T and of the main rotor thrust (Figure 24b). Two flows are encountered below the rotor: the induced flow, accelerated by the rotor, and the undisturbed flow created by descent of the helicopter. Meeting of these two flows leads to the onset of instability of the vortices, buffeting of the main rotor, and deterioration of control.

§ 17. Losses of the Real Rotor

We have been examining the operation of an ideal main rotor, i.e., a rotor in which all the power obtained from the engine was converted into work

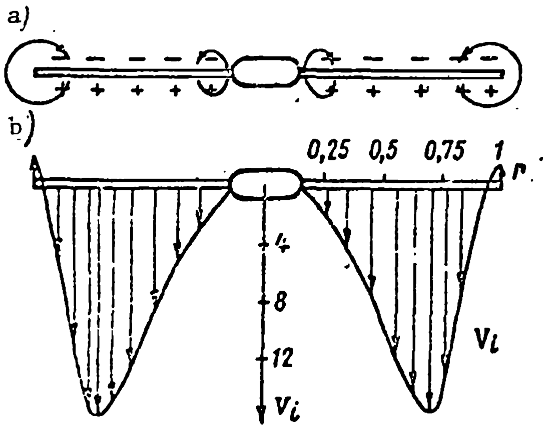

Figure 25. Main rotor losses.

in accelerating the air downward or in creating thrust.

We have assumed that the entire area swept by the rotor participates in creating thrust. This means that the increased air pressure below the rotor and the reduced air pressure above the rotor (Figure 25a) acts on the entire main rotor area. In reality, as will be shown later, the entire swept area does not participate in creating thrust. The ideal rotor accelerates a uniform air jet downward with the same induced velocity for all the blade elements. The real rotor provides a swirling jet, and the induced velocities will vary markedly along the radius for the different blade elements (Figure 25b).

The ideal rotor does not expend energy in overcoming friction forces, while the real rotor experiences profile drag forces resisting rotation,and considerable power is expended in overcoming these forces. Moreover, the real rotor has the so-called tip and root losses. The essence of these losses lies in cross-flow of the air from the high-pressure region below the rotor into the low-pressure region above the rotor. This cross-flow takes place through the ends of the blades (tip losses) and through the root sections of the blades near the main rotor hub (root losses), where the structural part of the blade (spar) does not have a lifting surface. The concept of the end loss coefficient χ has been introduced to account for the tip and root losses. With account for this coefficient, the actual area participating in creation of thrust is defined by the formula

$$F_e = F\chi.$$

For most main rotors $\chi = 0.90 - 0.92$. /33

Since for the real rotor V_i varies along the radius, we take as the induced velocity its value at the radius $\bar{r} = 0.7$

$$V_{i_{0,7}} \approx V_i.$$

To account for the influence of the profile drag forces, we assume that the real rotor power required for creating thrust is greater on the average than the ideal rotor power required by 25%.

With account for these losses, the thrust of the real rotor can be found from the formula

$$T = 2\chi F \rho V_i^2.$$

Hence, it is easy to find the induced velocity in the hovering regime

$$V_i = \sqrt{\frac{T}{2\chi F \rho}}.$$

Knowing that

$$T = C_T F \frac{\rho}{2} u^2,$$

we obtain

$$V_i = \sqrt{\frac{C_T F \rho u^2}{4\chi \rho F}} = \frac{u}{2} \sqrt{\frac{C_T}{\chi}}.$$

For most main rotors, the induced velocity in the hovering regime is $V_i \approx 8-10$ m/sec, and $C_T \approx 0.003$.

An important characteristic of the main rotor is the relative efficiency $\eta_0 = \frac{N_i}{N_{sup}}$.

The main rotor relative efficiency is the ratio of the power required to create the thrust of the ideal rotor to the total power supplied to the rotor. For modern rotors, the efficiency is 0.6-0.75.

§ 18. Characteristics of Operation of Coaxial System of Two Main Rotors

In the coaxial twin-rotor helicopter, the main rotors are positioned on a single axis — one above and the other below. Such a helicopter has certain operational characteristics. The area swept by the two main rotors is equal to the area swept by a single rotor

$$F_c = F_1$$

where F_c is the area swept by the system of coaxial rotors;
F_1 is the area swept by a single rotor.

In this case, we have assumed that the diameters of the upper and lower rotors are the same. /34

Let us examine the system of air jets passing through the areas swept by the upper and lower rotors (Figure 26). Increase of the distance between the hubs of the upper and lower rotors degrades the operating conditions of the lower rotor and complicates the construction of the entire system, while reduction of this distance leads to the danger of collision of the rotor blades and increases helicopter vibration. This distance is $h = 0.08D = 0.8m$ in the Ka-15 and Ka-18 helicopters. At this distance, the lower rotor has no effect on the operation of the upper rotor. The jet from the upper rotor contracts, and in the plane of rotation of the lower rotor its radius is 0.7R, where R is the rotor radius. In this case, the lower rotor blade tips

operate under the same conditions as those of the upper rotor and draw additional air in from the side.

On this basis, we shall estimate the effective area of the entire system through which the air flows, just as for an isolated rotor in the hovering regime.

From the area swept by the upper rotor, we must subtract the root loss area (of radius 0.25R). Under conditions similar to those in the hovering regime, only the tips of the lower rotor blades operate. The area swept by these tips is

$$F_1 = \pi R^2 - \pi 0.7^2 R^2.$$

Consequently, the effective area of both rotors through which the stream flows, as in the case of hovering of an isolated rotor, is found from the formula

$$F_c = \pi R^2 - \pi 0.25^2 R^2 + \pi R^2 - \pi 0.7^2 R^2 = \pi R^2 (1 - 0.06 + + 1 - 0.49) = 1.45 F_1.$$

That portion of the lower rotor which operates in the jet of the upper rotor has lower efficiency. The angles of attack of the lower rotor blade elements are reduced as a result of the induced velocity of the upper rotor (see Figure 23b), which leads to reduction of the thrust. To reduce this effect, the incidence angles of the lower rotor blades are made 2-3° larger than for the upper rotor, but this does not eliminate entirely the harmful influence of the upper rotor on the lower. In the presence of this influence, the efficiency of the central portion of the lower rotor, which is in the jet from the upper rotor is reduced by a factor of two, in comparison with the efficiency of the tip area outside the jet from the upper rotor.

The swept area of the lower rotor, operating in the jet from the upper rotor, is found from the formula /35

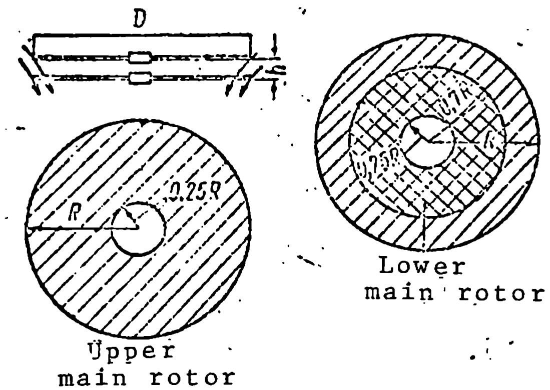

Figure 26. Operation of coaxial rotor system.

$$F_1 = \pi R^2 0{,}7^2 - \pi R^2 0.25^2 =$$
$$= \pi R^2 0.43 = 0.43 F_1.$$

Since its efficiency is less than that of the upper rotor by a factor of two, the additional effective area of the lower rotor is

$$F_{e.1} = 0.43 F_1 0.5 = 0.22 F_1.$$

The effective area of the entire system is $F_{e.1} = 1.45F_1 + 0.22F_1 = 1.67F_1$. This formula shows that the thrust of two coaxial rotors under the same conditions is greater than the thrust of an isolated main rotor of the same diameter by a factor of 1.67.

If the thrusts of the coaxial system and the isolated rotor are the same, then less power is required to create the thrust of the coaxial rotor system, which follows from ideal rotor momentum theory.

The power required to turn the ideal rotor is entirely converted into kinetic energy of the jet, i.e., $N_i = TV_i$.

If we use T_c, V_c, F_c, respectively, to denote the thrust, induced velocity, and effective area of the coaxial system of two rotors, and T_1, V_1, F_1 to denote the thrust, induced velocity, and swept area of the isolated rotor, then we have $T_c = T_1$.

Consequently,

$$2\rho F_c V_c^2 = 2\rho F_1 V_1^2.$$

We know that

$$F_c = 1.67F_1.$$

Then

$$2\rho 1.67F_1V_c^2 = 2\rho F_1V_1^2.$$

Hence, we find

$$V_c^2 = \frac{2\rho F_1V_1^2}{2\rho 1.67F_1} = \frac{V_1^2}{1.67},$$

or

$$V_c = \sqrt{\frac{V_1^2}{1.67}} \approx 0.78V_1.$$

In order to obtain thrust on a system of coaxial rotors equal to the thrust of an isolated rotor of the same diameter, the induced velocity of the coaxial system must be less than the induced velocity of the isolated rotor.

Since the ideal rotor power required is proportional to V_i, less power is required to obtain the same thrust for the coaxial system than for the /36 isolated rotor. This is the advantage of the coaxial system. The number 0.78 $\approx$ is called the aerodynamic advantage coefficient, and is denoted by ζ_A. Using this coefficient, we express the power required for the coaxial system in terms of the power required of an isolated ideal rotor

$$N_c = \zeta_A N_l = 0.8N_l.$$

This implies that for the same power the coaxial rotor system provides 13-15% more thrust than the isolated main rotor. Therefore, the helicopter with coaxial rotors has smaller dimensions than the single-rotor helicopter.

However, to date only light helicopters have been built using this scheme because of structural complexity and other problems.

Twin-rotor helicopters of other arrangements, for example, with the rotors placed longitudinally and with intermeshing rotors, also have an aerodynamic advantage in the axial flow regime. The aerodynamic advantage coefficient of these systems approaches closer to 0.8, the less the distance between the main rotor hub axes.

Programmed Testing Questions and Answers

In Chapters 1 and 2 we have examined concepts which are of considerable importance in themselves and ensure further successful study of helicopter aerodynamics. We shall present some questions and answers to test the readers' knowledge of this information.

The objective is to select the most complete and correct answer from three or four possibilities. Some of the answers given are completely incorrect, most of the answers are simply incomplete.

Question 1. Definition and purpose of blade geometric twist.

Answer 1. Geometric twist involves variation of the incidence angles of the blade elements. Twist is provided to distribute the loads uniformly over the blade and increase main rotor thrust.

Answer 2. Geometric twist involves variation of the blade element incidence angles along the main rotor radius. The root elements have larger incidence angles, and the tip elements have smaller angles. Twist gives the blade elements angles of attack close to the optimal values and increases the main rotor thrust by 5-7%. Twist results in more uniform loading on the individual blade elements and delays flow separation from the tip portion of the blade.

Answer 3. Geometric twist is the difference between the incidence angles at the root and tip sections of the blade. Twist provides minimal incidence angles at the root elements and maximal angles at the tip elements. This is necessary to obtain higher rotor efficiency, increase thrust, and achieve more uniform loading on the different parts of the blade.

Question 2. Main rotor operating regime coefficient. /37

Answer 1. The main rotor operating regime coefficient is the dimensionless number μ, equal to the ratio of the helicopter flight speed to the blade tip induced velocity

$$\mu = \frac{V}{V_i}.$$

Answer 2. The main rotor operating regime coefficient is the number μ, equal to the ratio of the projection of the flight velocity on the main rotor hub axis to the blade tip circumferential velocity

$$\mu = \frac{V \sin A}{\omega R}.$$

Answer 3. The main rotor operating regime coefficient is the number μ, equal to the ratio of the helicopter flight speed to the blade tip angular velocity

$$\mu = \frac{V}{\omega}.$$

Answer 4. The operating regime coefficient is the number μ, equal to the ratio of the projection of the helicopter flight speed on the main rotor hub plane of rotation to the blade tip circumferential velocity

$$\mu = \frac{V \cos A}{\omega R}$$

Question 3. What is the connection between the operating regime coefficient and the main rotor operating regime?

Answer 1. The larger μ, the larger the main rotor induced velocity and the closer its operating regime approaches the axial flow regime.

Answer 2. If $\mu = 0$, this indicates the axial flow regime. The larger μ, the more effectively the properties of the axial flow regime manifest themselves.

Answer 3. Increase of the coefficient μ indicates increase of the main rotor angle of attack and approach of its operating regime to the axial flow regime.

Question 4. What is the connection between the main rotor angle of attack and its operating regime?

Answer 1. The main rotor angle of attack is the angle between the flight velocity vector and the hub rotation plane. In the axial flow regime, the main rotor angle of attack $A = \pm 90°$, in the inclined flow regime $A \neq \pm 90°$.

Answer 2. The main rotor angle of attack is the angle between the flight velocity vector and the hub axis. If the main rotor angle of attack $A = 90°$, the rotor is operating in the axial flow regime. However, if $A \neq 90°$, it is operating in the inclined flow regime.

Answer 3. The main rotor angle of attack is the angle between the plane of rotation of the main rotor and the vector of the undisturbed flow approaching the rotor. For $A = 0°$ the inclined flow regime is present; for $A \neq 90°$ the flow regime is axial.

Question 5. What is main rotor thrust, and on what does it depend?

Answer 1. Main rotor thrust is the aerodynamic force which arises during rotor rotation as a result of the difference of the air pressure on the rotor blades

$$T = C_T F \frac{\rho}{2} u^2.$$

The thrust depends on the rotor area thrust coefficient, flight speed, and air density. The thrust coefficient depends on rotor rpm and blade element pitch. /38

Answer 2. Main rotor thrust is the aerodynamic force directed along the main rotor axis and formed as a result of the difference of the air pressures below and above the rotor

$$T = C_T F \frac{\rho}{2} R^2 \omega^2.$$

The thrust depends on the thrust coefficient, main rotor area or radius, air density, and main rotor rpm. The thrust coefficient depends on the pitch.

Answer 3. Main rotor thrust is the aerodynamic force which arises as a result of the difference of the air pressure below and above the rotor

$$T = C_T F \frac{\rho}{2} \omega R R \quad \text{or} \quad T = 2\rho F V_i^2.$$

The main rotor thrust depends on the thrust coefficient, area swept by the rotor, rotor pitch, and rotor rpm. The thrust increases with increase of the pitch and rpm.

Question 6. What is the main rotor reactive torque, what does it depend on, and how does it act?

Answer 1. The reactive torque is the torque opposing rotor rotation

$$M_r = Q_b R k$$

It retards rotor rotation and yaws the helicopter opposite the direction of rotation. The reactive torque depends on the rotor rpm, air density, rotor pitch, and flight speed.

Answer 2. The reactive torque is the moment of the forces of resistance to rotation about the hub axis. It is defined by the formula

$$M_r = Q_b r_Q k.$$

It depends on the rpm, pitch, air density, surface condition and flight speed. It retards rotor rotation and yaws the helicopter opposite the direction of rotor rotation.

Answer 3. Reactive torque is the moment of the forces of resistance to rotation, directed opposite the rotor direction of rotation, retarding rotor rotation and yawing the helicopter opposite the direction of rotation. It depends on the flight speed, rpm, and air density

$$M_r = 2Q_b rk.$$

Question 7. Power required to rotate the main rotor and the constant rpm conditions.

Answer 1. The power required to turn the main rotor depends on the rpm, pitch, flight speed, and air density

$$N_{req} = m_{tor} F \frac{\rho}{2} \omega^3 R^3.$$

If $N_{sup} = N_{req}$ the rpm is constant; if $N_{sup} > N_{req}$ the rpm increases.

Answer 2. The power used to overcome the reactive torque depends on the rotor pitch, rpm, and flight speed

$$N_{req} = M_r \omega.$$

When $N_{sup} = N_{req}$ the rpm remains constant, when $N_{sup} > N_{req}$ the rpm increases.

Answer 3. The power required to turn the main rotor and overcome the retarding action of the reactive torque depends on the main rotor thrust, rpm, air density, and flight speed /39

$$N_{req} = F \frac{\rho}{2} \omega^2 R^3 m_{tor}.$$

When $N_{sup} = N_{req}$ the rpm is constant; when $N_{sup} > N_{req}$ the rpm increases.

Question 8. What is the rotor blade element angle of attack and how is it changed?

Answer 1. The blade element angle of attack is the angle between the blade chord and the resultant velocity vector. It depends on the blade element pitch, induced velocity, and helicopter flight speed. The larger the induced velocity, the lower the angle of attack. The larger the vertical climbing velocity, the lower the angle of attack.

Answer 2. The blade element angle of attack is the angle between the blade element chord and the resultant velocity vector. It depends on the flight speed and induced flow downwash angle. With increase of the induced velocity, the angle of attack increases, with increase of the flight velocity it decreases.

Answer 3. The blade element angle of attack is the angle between the chord and the circumferential velocity vector. It depends on the pitch and helicopter flight speed. With increase of the vertical descent velocity, the angle of attack increases. With increase of the vertical climbing velocity, the angle of attack decreases.

CHAPTER IV

MAIN ROTOR OPERATION IN FORWARD FLIGHT

§ 19. Characteristics of Main Rotor Operation in Forward Flight

We recall that the term forward flight refers to operation of the main rotor in an undisturbed stream which approaches the rotor *nonparallel to the hub axis* (see Figure 12c). While in the axial flow case, the rotor imparts to the air mass traveling along the axis additional momentum in the same direction, in the case of forward flight the rotor also imparts to a definite air mass some additional momentum, only this time not in the direction of the undisturbed approaching stream, rather in the direction along the rotor axis, downward. This leads to the appearance of the so-called downwash (Figure 27a). The *downwash magnitude* is connected directly with the magnitude of the thrust which the main rotor develops in the forward flight regime.

In accordance with wing and propeller vortex theory, developed by Zhukovskiy in the 1905-1921 period, the wing lift and the main rotor thrust in the forward flight regime can be determined using the same formulas.

We imagine a stream of circular cross section, flowing past a wing (Figure 27b). The stream approaches the wing with the velocity V. As a result of the formation of the *induced* vortices, the wing imparts to the air /40
mass per second m_s the vertical velocity u, termed the induced velocity. Vortex theory shows and experimental aerodynamics confirms that there is a gradual increase of the induced velocity behind the wing.

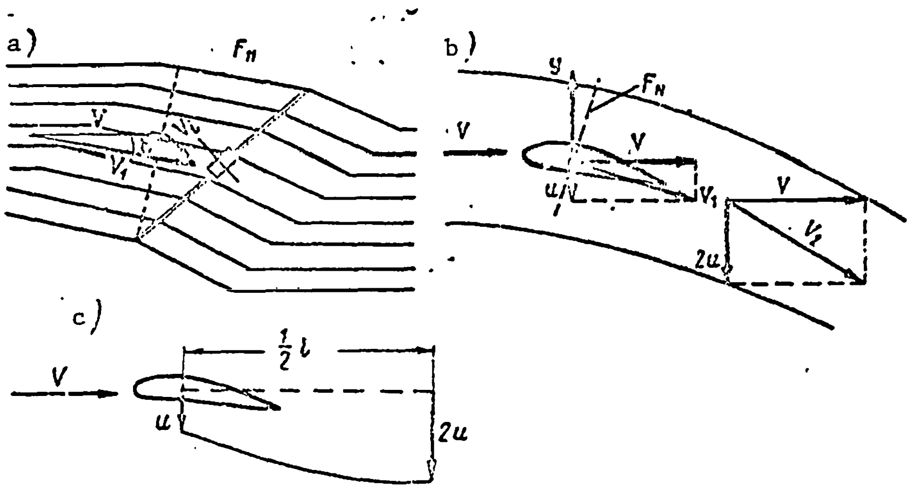

Figure 27. Operation of main rotor in forward flight regime.

At a distance equal to about $0.5l$ (wing half-span) the induced velocity reaches the value 2u (Figure 27c). Thus, the air acquires from the wing additional momentum equal to $m_s 2u$.

The energy conservation law states that the momentum increase equals the impulse of the force. The impulse of the force per second will be simply the wing lift. Consequently,

$$Y = m_s 2u. \tag{13}$$

Let us find the magnitude of the air mass flowrate m_s. The stream section area F_N, normal to the vector V_1, equals the area of a circle of diameter equal to the wingspan l.

$$F_N = \pi \frac{l^2}{4}.$$

The velocity vector $\overline{V}_1 = \overline{V} + \overline{u}$ (V is the undisturbed flow velocity, and u is the induced velocity). Then

$$m_S = \rho F_N V_1. \qquad (14)$$

Substituting this value of the mass flowrate into (13), we obtain

$$Y = 2\rho F_N V_1 u. \qquad (15)$$

Thus, the wing lift depends on the air density, wingspan, flight speed, and the induced velocity with which the wing deflects the stream downward.

From (15) we find the magnitude of the induced velocity /41

$$u = \frac{Y}{2\rho F_N V_1}.$$

Since the stream induced downwash angle is small, we can assume that $V_1 \approx V$.

The downwash formed by the main rotor (see Figure 27a) is similar to the downwash due to a wing with span l = D.

The air approaches the rotor with the velocity V and is deflected downward as a result of the induced inflow velocity V_i. The resultant rotor velocity will be equal to the vector sum of the velocities of the undisturbed stream and the induced velocity

$$\overline{V}_1 = \overline{V} + \overline{V}_i.$$

The angle ε between the vectors $\overline{V}$ and $\overline{V}_1$ is the induced downwash.

Continuing the comparison with the airplane wing, we can say that the air mass flowrate $\dot{m}_S = \rho F_N V_1$ passes through the area F_N normal to the resultant velocity vector $\vec{V}_1$. Since the rotor is taken to be a wing with span l = D, then

$$F_N = \pi \frac{l^2}{4} = \pi \frac{D^2}{4} = F,$$

i.e., the area perpendicular to the vector $\vec{V}_1$, will be equal to the area swept by the main rotor $F_N = F$.

In the forward flight regime the downwash velocity is also equal to twice the inflow velocity. On this basis and using ideal rotor momentum theory, we find the thrust in the forward flight regime using (4)

$$T = m_s V_{dw} = m_s 2V_i.$$

Using (14), we can write

$$T = \rho F_N V_1 2V_i.$$

If $F_N = F$, then

$$T = 2\rho F V_1 V_i.$$

If we account for tip and root losses, this formula can be written in the form

$$T = 2\chi\rho F V_1 V_i.$$

Consequently, main rotor thrust in the forward flight regime depends on air density, rotor pitch, and flight velocity.

§ 20. Main Rotor Thrust as a Function of Flight Speed /42

The thrust of a particular rotor at constant air density depends on the flight speed and the induced velocity. With increase of the flight speed there is an increase of the resultant velocity, which leads to increase of

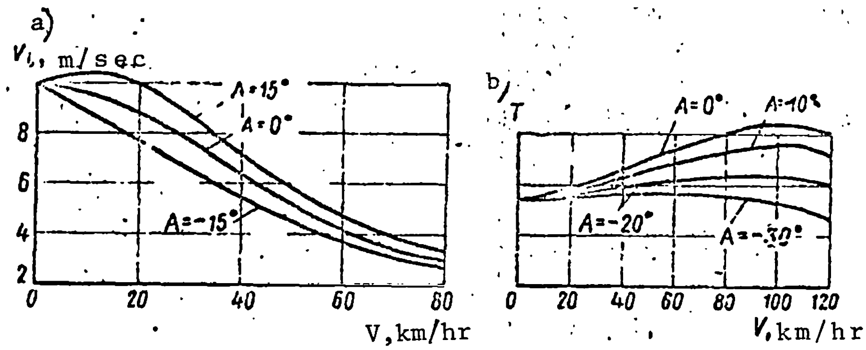

Figure 28. Main rotor induced velocity and thrust versus speed flight.

the mass flowrate of the air deflected by the main rotor. Consequently, the higher the velocity in the forward flight regime, the larger the air mass flowrate and the greater the thrust developed by the rotor. But thrust increase is possible only up to some limit. This is associated with the change of the induced velocity which, in turn, depends on the flight speed. However, this relation is complicated by the variation of the main rotor angle of attack (Figure 28a). This figure makes it possible to draw some important conclusions:

the induced velocity decreases with increase of the flight speed;

with increase of the main rotor angle of attack the induced velocity increases and vice versa;

for negative angles of attack the induced velocity decreases with increase of the flight speed;

for A > 0° the induced velocity first increases with increase of the flight speed up to 15-20 km/hr and then decreases;

for flight speeds up to 50-60 km/hr the induced velocity depends to a considerable degree on the main rotor angle of attack, while at higher

flight speeds this dependence becomes less significant;

the induced velocity decreases very rapidly with flight speed in the range from 0 to 60-70 km/hr.

With further increase of the flight speed, the reduction of the induced velocity becomes more gradual.

These conclusions are necessary for understanding the nature of main rotor thrust variation in the forward flight regime, and also for understanding the nature of helicopter motion in horizontal flight, climb, and descent along an inclined trajectory. If we take into account the nature /43
of the induced velocity variation, then the variation of main rotor thrust with change of the flight speed becomes clear (Figure 28b). This figure shows that main rotor thrust increases with increase of the flight speed and reaches the maximal value for a speed of about 100 km/hr. All the conclusions drawn on the variation of the induced velocity and thrust relate to operation of a main rotor with constant power expended in turning the rotor.

The thrust increase with increase of the flight speed is explained by the fact that, as the flight speed increases, a larger amount of air approaches the rotor, i.e., the mass flowrate of the air interacting with the rotor increases. The rotor deflects the large air mass downward and, thus, force impulse increases, i.e., the main rotor thrust increases.

Upon further increase of the flight speed, the time of interaction of the rotor with the air diminishes. The rotor "fails to" deflect the air markedly downward, which means a decrease of the induced velocity and, therefore, of the force impulse. Moreover, the energy received by the rotor from the shaft is expended not only in creating the induced velocity, but also in overcoming frictional drag forces, and with increase of the flight velocity these forces increase.

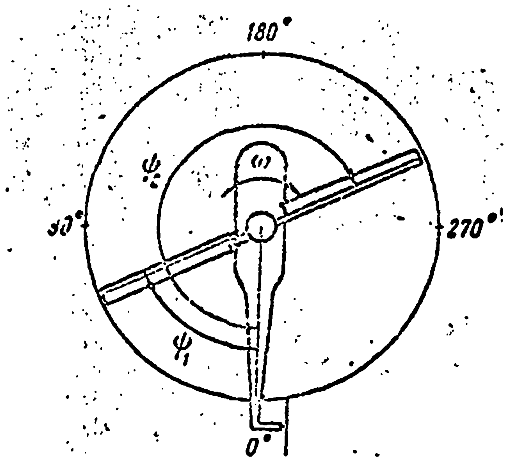

Figure 29. Blade azimuth.

§ 21. Blade Thrust and Its Azimuthal Variation

During rotation of the main rotor in the forward flight regime, there is continuous variation of the blade position relative to the flight velocity vector or the velocity vector of the undisturbed flow approaching the main rotor. This situation influences the nature of the flow over the blade and the forces which arise. The variation of the blade position is the reason for many phenomena which arise in the forward flight regime. A special concept is introduced to define this position — blade azimuth.

The azimuth, or the angle of the azimuthal position of the blade, is the angle between a reference line and the blade longitudinal axis at a given moment of time (Figure 29).

It is customary to take as the reference line the blade longitudinal axis when the blade is positioned directly aft of the main rotor hub.

The azimuth is reckoned from 0 to 360° in the direction of rotation of the main rotor and is represented by the letter ψ. The blade traveling from /44
from the 0° azimuth to the 180° azimuth is called the advancing blade. The blade travelling from the 180° azimuth to the 360° azimuth is called the retreating blade. The concepts of "advancing" and "retreating" blades are associated with the variation of the direction of the undisturbed stream approaching the blade.

In the case of the advancing blade, the undisturbed flow created by helicopter flight is directed at some angle to the blade leading edge, while

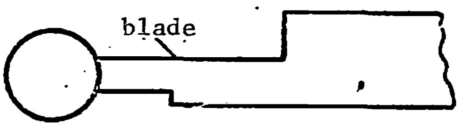

Figure 30. Blade root cutout.

in the case of the retreating blade, this flow is directed at the trailing edge, increases the vibration of the main rotor and the variation of the blade thrust as a function of azimuth, and reduces the main rotor thrust in the forward flight regime. The main rotor blade has a section of the lifting surface removed in the root region in order to reduce these undesirable phenomena (Figure 30). Increase of the cutout reduces the influence of reverse flow, but increases the root losses and, consequently, the magnitude of the thrust in the forward flight regime. An optimal size of the root cutouts is established for each main rotor.

The thrust of an individual blade can be found from the same formula used to obtain wing lift

$$T_b = C_{T_b} S_b \frac{\rho}{2} W^2,$$

where C_{T_b} is the blade thrust coefficient;

S_b is the blade planform area;

W is the resultant blade tip velocity.

The blade thrust coefficient depends on its shape and incidence angle; consequently, for a fixed pitch ϕ the quantities C_{T_b} and S_b are constants. Then for constant air density the blade thrust will vary similarly to the variation of the resultant velocity over the blade.

In the forward flight regime the blade thrust reaches its maximal value at the 90° azimuth, since in this case the resultant velocity over the blade is maximal. Conversely, at ψ = 270° the blade thrust is minimal, since the resultant velocity is least at this azimuth (see Figure 31c).

§ 22. Resultant Flow Velocity over Blade Element in the Hub Rotation Plane

It is well known that in the vertical flight regime each blade element is in a stream whose velocity is equal to the circumferential velocity of the element $u = \omega r$.

The situation is different in the forward flight regime. If the main rotor angle of attack $A = 0°$, the resultant velocity with which the stream flows over the blade element depends on the element circumferential velocity, the flight speed, and the azimuth angle ψ. In this case the resultant velocity will not be equal to the geometric sum of the circumferential velocity and the flight velocity, since only the flow directed perpendicular to the blade longitudinal axis has an influence on the aerodynamic forces of the element. /45

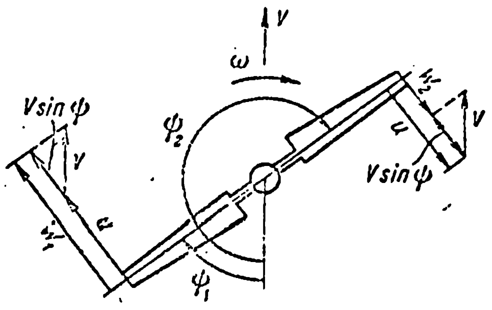

Figure 31. Blade element resultant velocity in forward flight regime.

Therefore, we must take as the resultant blade element velocity in the forward flight regime the sum of the vectors of the circumferential velocity of the blade element and the projection of the flight velocity vector on the line of the circumferential velocity vector (Figure 31).

$$\overline{W} = \overline{u} + \overline{V}\sin\psi. \tag{16}$$

Consequently, for a constant flight speed and constant angular velocity the resultant velocity will vary as a function of the azimuth angle.

Let us examine the variation of the resultant velocity as a function of blade azimuth (Figure 32).

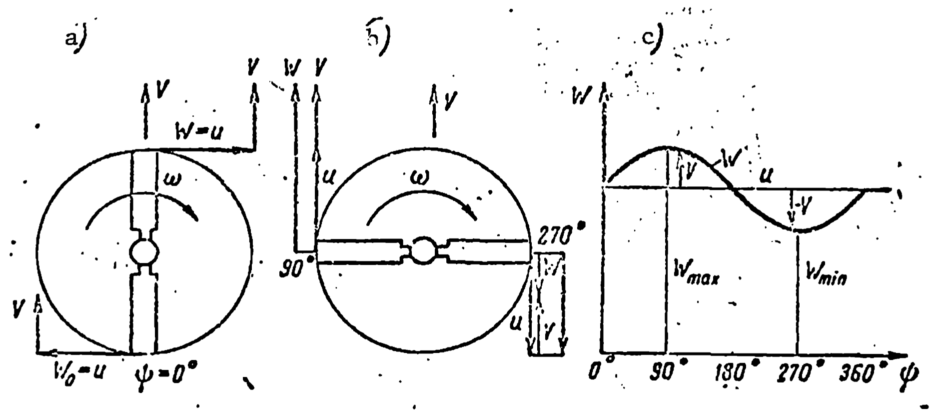

Figure 32. Blade element resultant velocity as a function of azimuth.

It is not difficult to see that for ψ = 0° and 180° the resultant velocity equals the circumferential velocity, since the projection of the flight velocity on the circumferential velocity vector equals zero (Figure 32a)

$$W_0 = u + V\sin 0° = u,$$
$$W_{180} = u + V\sin 180° = u.$$

For ψ = 90° the resultant velocity is /46

$$W_{90} = u + V\sin 90° = u + V.$$

For ψ = 270° the resultant velocity equals the difference of the velocities (Figure 32b)

$$W_{270} = u + V\sin 270° = u - V.$$

If we use (16) to calculate the resultant velocity for several azimuths, we can plot the relation W = f(ψ) (Figure 32c).

Figure 32 makes it possible to conclude that:

the maximal blade element velocity will occur at $\psi = 90°$, the minimal will occur at $\psi = 270°$; for $\psi = 0°$ and $180°$ the resultant velocities of a given element are equal to the circumferential velocity of this element. Consequently, the forward flight regime differs from the vertical flight regime in the variation of the blade element velocity. In the vertical flight regime this velocity remains constant $W = u$ and is independent of the azimuth. In this regime the "blade azimuth" concept has no meaning. In the forward flight regime the resultant blade element velocity in the hub rotation plane varies continuously.

§ 23. Variation of Circumferential and Resultant Velocities along Main Rotor Radius

Let us examine the velocity diagram of different blade elements of a two-bladed rotor when the blades are at the 90° - 270° azimuths. We shall consider the vectors of the reversed flow, i.e., the velocity vectors of the flow which approaches the blade element as a result of the circumferential velocity and the flight velocity. The velocity vectors of the motion of a point on the blade element were shown previously in Figure 32a, b.

In the diagram of Figure 33 we examine the reversed vectors. We see the following from the figure.

1. The circumferential velocities increase from zero at the hub axis. The variation of the circumferential velocities of the various elements up to the maximal value at the tip elements is shown along the line OD or OE.

2. All the blade elements travel with the velocity of the helicopter. If we draw the line FG parallel to the line ED at the distance V, we obtain the diagram of the resultant velocities of the various elements.

3. At the 90° azimuth the resultant velocity of all the elements is $u + V = \omega r + V$; at the point O, $W = V$.

4. At the 270° azimuth the resultant velocity is $\omega r - V$.

5. At the outboard blade elements, located between points A and C, the circumferential velocity is greater than the flight velocity and, consequently, the difference $u - V$ is positive, i.e., $W > 0$. Therefore, the air flow approaches the outboard elements from the leading edge. There is direct flow over the blade elements just as at the 90° azimuth, but with a lower velocity. /47

6. The blade element located at the point A has a circumferential velocity equal to the flight velocity $u = V$. Since these velocities are directed oppositely, the resultant velocity of this element is zero.

7. For the inboard elements between points A and 0 the circumferential velocity is less than the flight velocity ($u < V$), i.e., for these elements the difference $u - V = - W$. This means that the flow approaches these elements from the trailing edge. There is reversed flow over the inboard elements at azimuths close to 270°.

8. The reversed flow zone has the diameter d, which can be determined from the similar triangles ODC and OBA. In these triangles $OC = R$; $OA = d$; $CD = u = \omega R$; $AB = V$. From the basic property of similar triangles

$$\frac{OA}{OC} = \frac{AB}{CD}, \quad \text{or}$$

$$\frac{d}{R} = \frac{V}{\omega R},$$

hence

$$d = R\frac{V}{\omega R}.$$

Knowing that

$$\frac{V}{\omega R} = \mu,$$

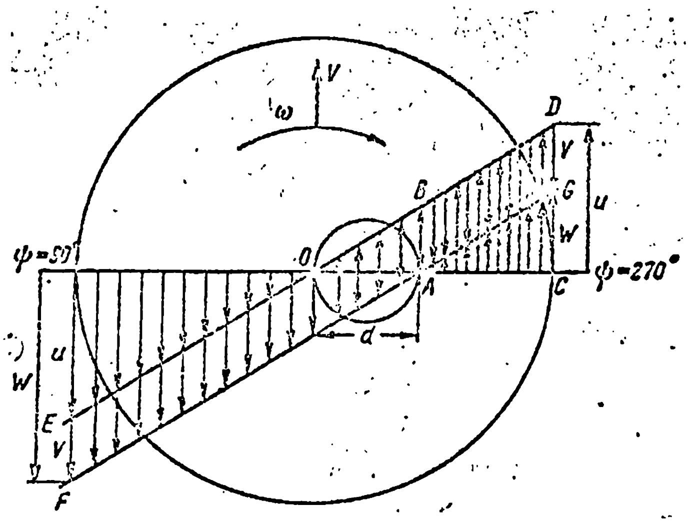

Figure 33. Circumferential and resultant velocities of different blade elements.

we finally have d = Rμ.

Conclusion: the diameter of the reverse flow zone is larger, the larger the main rotor radius and the larger the main rotor operating regime coefficient μ, i.e., the higher the flight velocity for a given rotor rpm. /48

As a result of the reverse flow, negative thrust develops on the portion of the blade located in this zone; this negative thrust is reduced by the blade root cutout.

§ 24. Drawbacks of Main Rotor with Rigid Blade Retention

The main rotors of the early helicopters (TsAGI1-EA, for example) had blades which were rigidly attached to the hub. The blade incidence angle was changed by means of axial hinges. In their arrangement such rotors are similar to airplane variable pitch propellers. But the very first flights disclosed major deficiencies characteristic of these rotors.

The thrust is created by all the blade elements, but the highest elemental forces will be those on the elements located at 3R/4 (see Figure 15d). The resultant of the elemental forces is applied at the blade center of pressure, which is located at the element with relative radius $\bar{r} = 0.7$. This distribution of the elemental thrust forces and this positioning of the resultant leads to the creation of a large bending moment at the blade root (Figure 34a). The approximate magnitude of the blade root bending moment at the blade

attachment to the hub is determined from the formula $M_{bend} = T_b\ 0.7R$.

Thus, if the rotor has four blades and the helicopter flight weight is 6000 kgf, the thrust of a single blade will be T_b = 6000:4 = 1500 kgf. For main rotor diameter D = 20 m, M_{bend} = 1500 x 0.7 x 10 = 10,5000 kgf·m. This moment will be still larger for a heavy helicopter. The large bending moment creates a large load on the blade root. Moreover, the blade is subjected to a centrifugal force which reaches a magnitude of several tens of tons; consequently the root portion of the blade operates under conditions of large loads. In order to avoid blade failure,the area of its root section must be increased, and this leads to increase of the structural weight and reduction of the helicopter's useful load.

Since the blade thrust varies azimuthally, its bending moment also varies (Figure 34b). The variable bending causes fatigue stresses in the material of the structural elements, which can lead to rapid blade failure. The up and down bending vibrations of the blade tips reach high frequencies (up to 3 - 4 cycles per second), creating heavy vibration of the helicopter.

The blade thrust does not vary azimuthally in the vertical flight regime, and this means that the main rotor thrust vector, equal to the sum of the blade thrust forces $T = T_b k$, lies along the hub axis (Figure 34c).

In the forward flight regime the blade thrust depends on the azimuth. /49
The thrust is maximal at the 90° azimuth and minimal at the 270° azimuth (Figure 34d). As a result of this variation, half the main rotor disk (advancing blades) has a higher thrust than the other half, formed by the retreating blades.

In this case the main rotor thrust vector T does not pass through the center of the hub, but rather at the distance a from the hub axis. The thrust moment M_T = Ta is created relative to the hub axis.

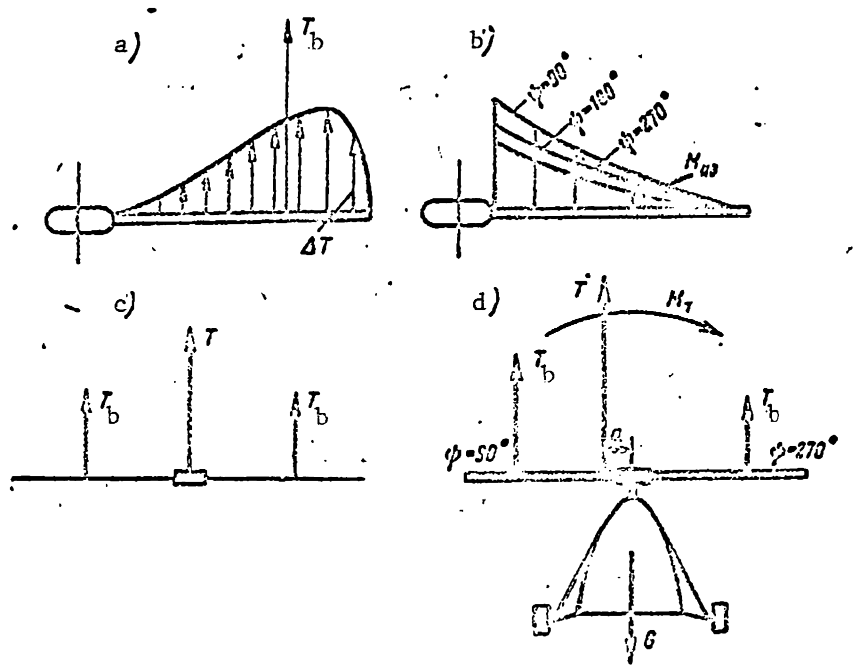

Figure 34. Blade bending moment and main rotor thrust overturning moment.

Since the hub axis is in the helicopter plane of symmetry, the main rotor thrust moment causes the entire helicopter to tend to overturn. This is termed the overturning moment. Thus the main rotor with rigid blade restraint has three major drawbacks:

presence of the overturning thrust moment in the forward flight regime;
presence of large thrust bending moment at the blade root;
variation of the blade thrust moment azimuthally.

All these drawbacks can be eliminated if the blades are attached to the hub by means of horizontal hinges.

§ 25. Purpose of Main Rotor Hub Horizontal Hinges

The horizontal hinge (HH) has its axis in the plane of rotation of the hub, perpendicular to the longitudinal axis of the blade (Figure 35a). The blade thrust develops a moment which rotates the blade about this hinge. The thrust moment $M_T = Ta$ causes rotation of the blade relative to this hinge, /50 and this means that the moment is not transmitted to the hub (the thrust overturning moment is eliminated) (Figure 35b).

When the horizontal hinge is used, the thrust-force bending moment at the root of the blade becomes zero, thus unloading the root section; the blade bending is reduced and therefore blade fatigue stresses are reduced and blade service life is increased. The vibrations caused by azimuthal variation of the blade thrust-force moment are also reduced. Summarizing, we can say that the horizontal hinges are intended to:

eliminate the main rotor thrust overturning moment in the forward flight regime;

relieve the blade root section of the thrust bending moment;

reduce fatigue stresses in the blade and vibrations caused by azimuthal variation of the blade thrust moment.

In addition, the horizontal hinges simplify control of the main rotor and helicopter, improve helicopter static stability, and reduce the magnitude of the azimuthal blade thrust variation.

§ 26. Conditions for Blade Equilibrium Relative to the Horizontal Hinge

Let us examine the blade thrust moment relative to the horizontal hinge. If this moment is not transmitted to the hub but simply rotates the blade, then a question arises immediately: how is the blade thrust transmitted through the hinge to the hub? In order to answer this question we examine

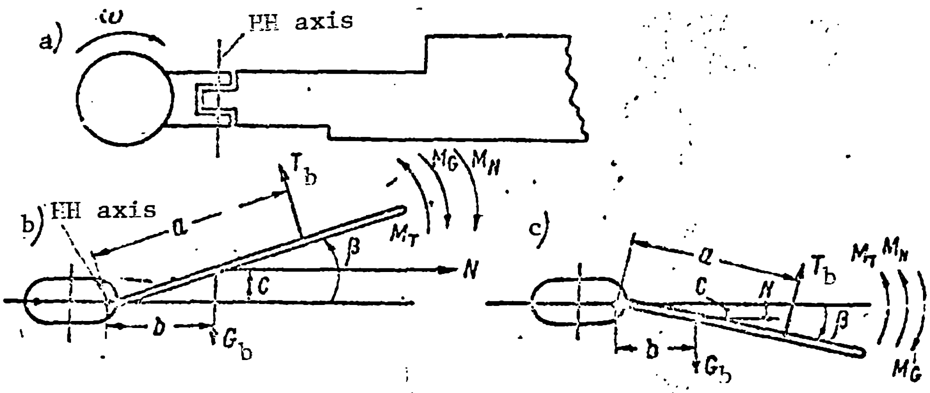

Figure 35. Blade equilibrium about horizontal hinge.

the conditions for blade equilibrium relative to the horizontal hinge.

In addition to the thrust force, in the plane perpendicular to the hub rotation plane there act the weight force G_b and the centrifugal force N (Figure 35b). /51

Each of these forces develops a moment relative to the horizontal hinge. The blade thrust moment rotates the blade upward. The flapping angle β is formed between the blade longitudinal axis and the hub rotation plane. When the blade tip is above the hub rotation plane,the blade flapping angle is considered positive.

The thrust moment rotates the blade in the direction of increasing flapping angle (blade flaps upward). The weight moment $M_G = G_b b$ rotates the blade downward, reducing the flapping angle. The centrifugal force moment rotates the blade to bring it closer to the hub rotation plane. If the flapping angle is positive, the centrifugal force moment $M_N = Nc$ rotates the blade downward and coincides in direction with the blade weight force moment. If the flapping angle is negative (Figure 35c), the centrifugal force moment rotates the blade upward and coincides in direction with the thrust force moment. Thus the centrifugal force moment tends to reduce the deflection of the blade from the hub rotation plane.

The centrifugal force always acts in the plane of rotation, is directed outward from the axis, and is applied to the blade center of gravity. It is defined by the formula

$$N = \frac{mu^2}{r} = \frac{G_b u^2}{gr} = \frac{G_b}{g} \omega^2 r.$$

The blade centrifugal force of the Mi-4 helicopter at maximal main rotor rpm exceeds 20,000 kgf. Therefore, even with a small arm, c the moment of this force will be very large.

After seeing what moments act on the blade about the horizontal hinge, we can define the equilibrium condition

$$\sum M_{HH} = 0.$$

This condition can be written as follows for positive and negative flapping angles:

for β > 0

$$M_T = M_G + M_N \quad \text{or} \quad Ta = G_b b + N_c\,; \tag{17}$$

for β < 0

$$M_G = M_T + M_N.$$

Equilibrium in the case of negative flapping angles is possible, but only in the course of a very limited time. Therefore in the following we shall consider (17) to be the equilibrium condition.

If this condition is violated, the blade will rotate until equilibrium is restored at a new flapping angle. With change of the flapping angle, there will be a change of the centrifugal force arm and therefore of its moment. /52

Thus the blade will flap upward if the thrust force moment is *greater* than the sum of the moments of the centrifugal force and the weight force, i.e., for $M_T > M_G + M_N$. But with increase of the flapping angle, the moment $M_N = Nc$ will increase and equilibrium will again be established. The same process will take place upon reduction of the flapping angle, but in the reverse direction.

The flapping angle has a comparatively small value — 7 - 10°.

The primary reason for violation of blade equilibrium relative to the horizontal hinge is the variation of the blade thrust and its moment.

The horizontal hinges have snubbers (stops) to limit the blade upward and downward rotation. The lower stop is the blade droop limiter, i.e., the blade rests on this stop if the rotor is not turning, which prevents the blade coming into contact with other parts of the helicopter. The stop has a centrifugal regulator which allows the blade to deflect to negative flapping angles in flight.

The upper stop limits the upward rotation of the blade (flapping angle 25 - 30°). The blade does not reach the limiters in flight, since the centrifugal force moment does not permit the blade to deflect very far from the hub rotation plane.

§ 27. Main Rotor Cone of Revolution

When horizontal hinges are used, the rotating rotor forms a "cone of revolution". If the flapping angle is positive, as the main rotor turns the blades travel along the generator of a cone whose apex is located at the center of the hub. The plane passing through the rotating rotor blade tips is called the main rotor plane of rotation [tip path plane] (Figure 36).

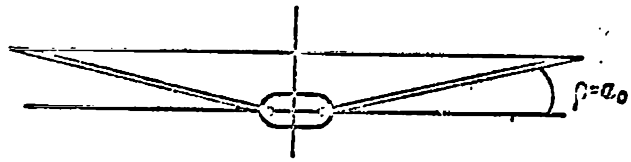

Figure 36. Main rotor coning.

The line passing through the center of the hub perpendicular to the tip path plane is called the coning axis. The main rotor thrust vector passes lies along the coning axis.

If the blade flapping angle does not change azimuthally, the main rotor tip path plane is parallel to the hub rotation plane and the coning axis coincides with the main rotor hub axis. In this case the flapping angle β equals the coning angle a_0. The coning angle is the angle a_0 between the hub rotation plane and the generator of the cone of revolution. It varies from 0° to 10 - 12°.

The coning angle is larger, the larger the main rotor pitch. In- /53
crease of the pitch leads to increase of the blade thrust and moment about the horizontal hinge.

The main rotor cone of revolution is clearly visible if all the blades have the same flapping angle, i.e., there exists so-called co-conicity of the blades, absence of which leads to severe vibration of the main rotor. To obtain co-conicity it is necessary that the incidence angles of all the blades be the same. The technique for adjusting the rotor for blade tracking is given in the helicopter maintenance instructions.

§ 28. Blade Flapping Motions

Blade motions relative to the main rotor hub horizontal hinges in the forward flight regime are called flapping motions. These motions arise when the blade equilibrium relative to the horizontal hinges is disrupted because of azimuthal variation of the blade thrust.

When the blade thrust and moment increase it flaps upward, and when the thrust and moment decrease it flaps downward. Let us see how the blade flapping

angles vary with azimuth.

For the advancing blade with ψ from 0 to 90° the resultant flow velocity over the blade and the blade thrust and moment increase, and the blade flaps upward — the flapping angle and the vertical velocity increase. At the 90° azimuth the upward vertical flapping velocity reaches the maximal value. For $\psi > 90°$ the blade thrust and vertical flapping velocity decrease, while the flapping angle continues to increase.

The blade flapping motions are affected not only by variation of the resultant velocity, but also by variation of the blade element angle of attack caused by the main rotor coning angle. As a result of the coning angle the undisturbed stream approaches the blade located at the 180° azimuth at some angle from below, and approaches the blade located at the 360° azimuth at some angle from above (Figure 37a).

The undistrubed stream velocity vector can be broken down into the components: $\bar{V}_y$ perpendicular to the blade longitudinal axis, and $\bar{V}_s$ parallel to the blade axis. The latter is called the slip velocity. The blade element angle of attack and thrust are independent of V_s. At the 180° azimuth the vector V_y is directed at the blade from below, consequently this leads to increase of the blade element angle of attack by the magnitude $\Delta\alpha$ (Figure 37b). The induced flow velocity is not shown in the figure.

At the 360° azimuth (Figure 37a) the vector V_y is directed downward toward the blade, which leads to reduction of the blade element angle of attack (Figure 37d). Thus, as a result of coning the angle of attack of each blade element changes azimuthally from a maximum at the 180° azimuth to a minimum at the 360° azimuth. At the 90° and 270° azimuths, the angles of /54 attack equal the incidence angle (without account for the induced velocity and the flapping motion velocity), Figure 37c.

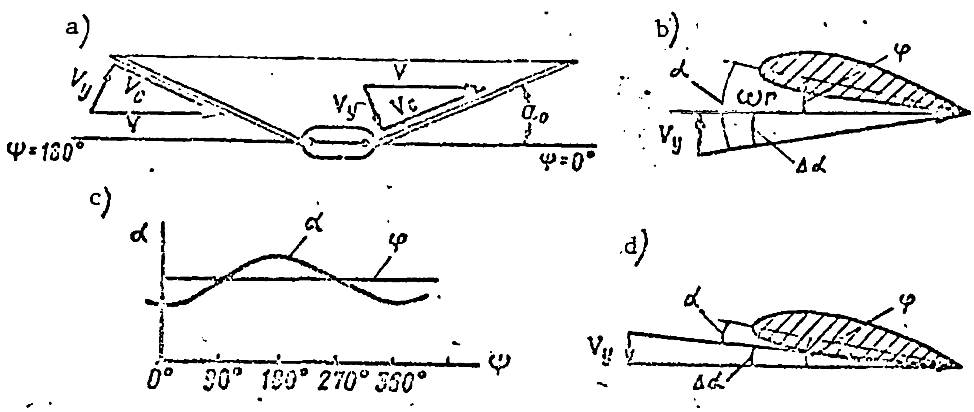

Figure 37. Blade element angle of attack as a function of main rotor coning angle.

But the increase of the blade element angle of attack as a result of the coning angle for $\psi > 90°$ leads to increase of the thrust and further upward flapping. As a result of this effect,the maximal blade flapping angle in the forward flight regime will occur at $\psi \approx 210°$. In this case equilibrium of the blade relative to the horizontal hinge is established. As the blade motion continues around the circle, the blade thrust decreases as a result of reduction of the resultant velocity and the blade element angle of attack, and equilibrium is disrupted, i.e.,

$$M_T < M_0 + M_N.$$

The vertical downward flapping velocity will be maximal at the 270° azimuth. Equilibrium is reached again for $\psi \approx 30°$ and the flapping angle will be minimal.

This variation of the flapping angle in azimuth is possible in the forward flight regime if the blade incidence angle does not change in azimuth and account is not taken of elastic twisting of the blade under the influence of the aerodynamic forces.

§ 29. Main Rotor Coning Axis Tilt

With variation of the flapping angles the plane of rotation and the coning axis deflect backward and to the side in the direction of the advancing blade through the angle τ (Figure 38a). As a result of the tilt of the coning axis backward by the angle a_1, there is an increase of the blade flapping angle to $\beta = a_0 + a_1$ at the 180° azimuth and a reduction to $\beta = a_0 - a_1$ at the 0° azimuth (Figure 38b). Tilting of the cone axis to the side by the angle b_1 leads to change of the flapping angles: at the azimuth 90° $\beta = a_0 - b_1$; at the azimuth 270° $\beta = a_0 + b_1$ (Figure 38c).

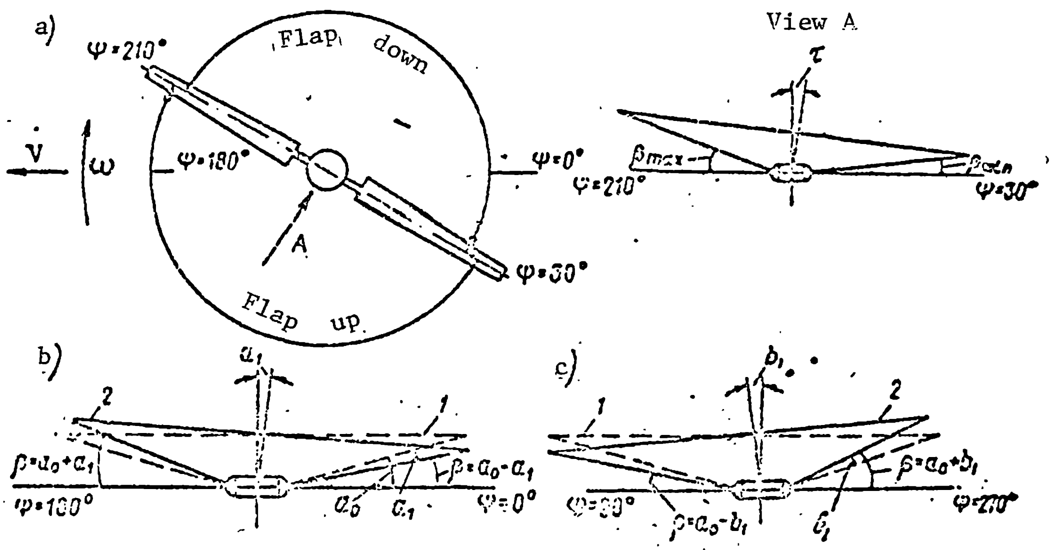

Figure 38. Blade flapping motions and tilt of main rotor cone axis. 1 - cone for $\mu = 0$; 2 - cone for $\mu > 0$.

Tilting of the cone axis backward by the angle a_1 leads to deflection /55
through the same angle of the thrust vector and the formation of the longitudinal thrust component H (Figure 39a). This force is the projection of the main rotor thrust on the hub rotation plane. Since it is directed aft, it is a drag force and is analogous to the induced drag of an airplane wing. The

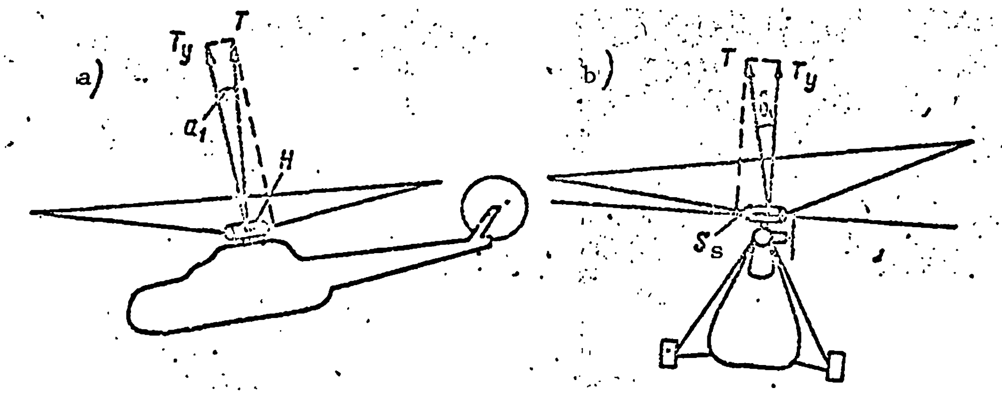

Figure 39. Main rotor thrust force components.

larger the flapping motions, the larger the backward tilt of the cone axis and the larger the longitudinal force H resisting helicopter forward motion. Consequently, the flapping motions in the forward flight regime must be restricted.

If the deflected thrust T is projected on the hub axis, we obtain the force required for helicopter flight

$$T_y = T \cos a_1.$$

In view of the smallness of the angle a_1(2 - 3°) we can take $a_1 \approx 1$. /56
Then $T_y \approx T$.

The sideward tilt of the cone axis (Figure 39b) leads to the appearance of the side force S_s, which is the projection of the main rotor thrust on the hub rotation plane

$$S_s = T \sin b_1.$$

Since this force is directed to the left, this direction is unfavorable for single-rotor helicopters. Therefore, the blade flapping motions must be restricted in order to alter the sideward tilt of the cone angle from the

left to the right. Moreover, restriction of the flapping motions is also necessary to reduce main rotor vibrations.

§ 30. Blade Flapping Motion Restriction and Flapping Compensator

The blade flapping motions are limited by the action of the centrifugal force moment. Moreover, the flapping motions themselves create aerodynamic limiting of these motions. The essence of the limiting amounts to the following. As the blade flaps upward (Figure 40a), the blade element angle of attack is reduced by $-\Delta\alpha$ as a result of the vertical flapping velocity V_{fl}, which leads to reduction of the blade thrust and moment and, consequently, to more rapid restoration of equilibrium about the horizontal hinge. When the blade flaps downward (Figure 40b), the angle of attack increases, which leads to increase of the thrust and limitation of the downward flapping motion.

But the restriction of the flapping motions as a result of centrifugal forces and aerodynamic limiting is not sufficient. Therefore use is made of the so-called pitch control arm compensation or flapping compensator.

The essence of the flapping compensator lies in a special positioning of the blade pitch control elements. It was established earlier that the blade pitch (incidence angle ϕ) changes with rotation of the blade about its longitudinal axis. Blade rotation is accomplished with the aid of the axial hinge, on the body of which there is the "blade pitch horn" lever. The /57
vertical rod from the main rotor tilt control is connected to the blade horn arm. Connection of the cyclic control rod with the blade horn is accomplished by means of the pitch horn hinge.

If the tilt control rod moves upward, the blade incidence angle is increased (Figure 41a).

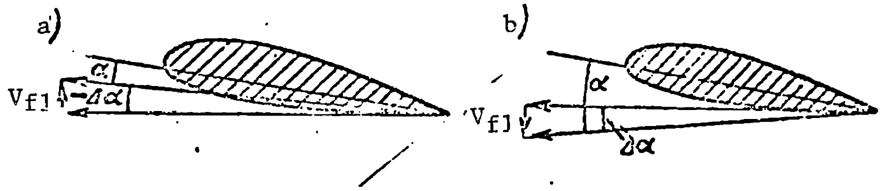

Figure 40. Blade element angle of attack change.

If the control rod moves downward, the blade incidence angle is reduced (Figure 41b). The location of the horn arm hinge relative to the main rotor hub horizontal hinge is of fundamental importance. It may be located on the axis of the horizontal hinge (Figure 41c) or it may be shifted relative to this axis by the distance a (Figure 41d).

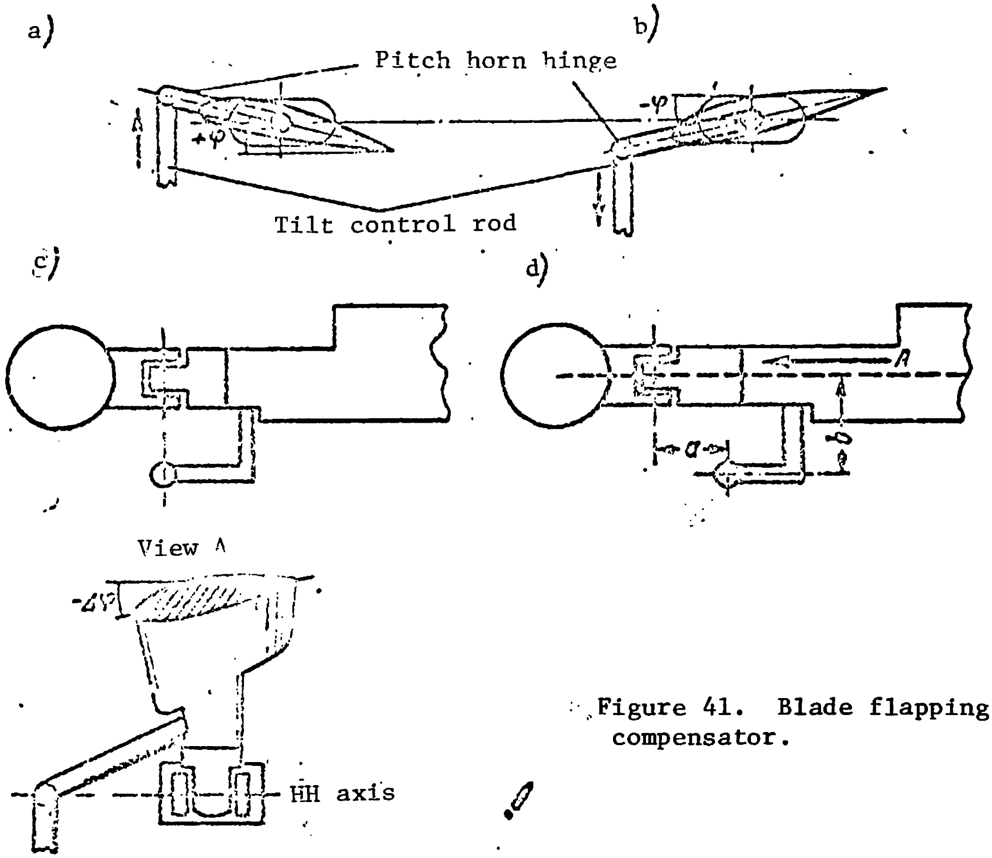

Figure 41. Blade flapping compensator.

In the first case, during flapping motion the horn rotates together with the blade about the horizontal hinge axis so that it does not hinder blade rotation. The blade chord displaces parallel to itself and the incidence angle remains constant.

In the second case, the shift of the horn hinge relative to the horizontal hinge axis leads to change of blade pitch during flapping motions. Thus, when the blade flaps upward the horn hinge, remaining stationary, holds back /58 the blade leading edge, i.e., it causes reduction of the pitch (Figure 41d). When the blade flaps downward its pitch increases. This sort of pitch change leads to limiting of the flapping motions. For example, when the blade moves upward the blade pitch is reduced and its thrust and moment are also reduced. Therefore, equilibrium is restored more rapidly and the flapping angle is reduced.

When the blade flaps downward, the increased pitch leads to increased thrust and the flapping angle is restricted. The effectiveness of the flapping compensator action depends on the flapping compensation coefficient K.

The compensation coefficient is the ratio of the distance a between the pitch horn hinge and the horizontal hinge axis to the distance b between the pitch horn hinge and the blade longitudinal axis (Figure 41d).

$$K = \frac{a}{b}.$$

The larger the compensation coefficient, the larger the blade pitch change with variation of the flapping angle and, consequently, the more the blade up and down flapping is restricted.

For most helicopters the compensation coefficient is about 0.5.

By increasing the compensation coefficient we can limit the increase of

the flapping angle for the advancing blade to a point where maximum flapping will not occur at the $\psi = 210°$ azimuth, as we have noted above, but rather at the $\psi \approx 160°$ azimuth. In this case, the minimal flapping angle of the retreating blade will occur at the $\psi \approx 340°$ azimuth. With this change of the flapping angles, the main rotor cone axis will be deflected aft and in the direction of the retreating blade, and the side force will be directed to the right.

§ 31. Blade Element Angle of Attack Change Owing to Flapping Motions

The blade angle of attack change $\pm\Delta\alpha$ depends on the vertical flapping velocity $\pm V_{fl}$, on ωr, and on $V \sin \psi$, i.e., on the azimuth angle, which we see from the formula

$$\mathrm{tg}\ \Delta\alpha = \frac{\pm V_{fl}}{\omega r + V \sin \psi}. \tag{17a}$$

The sign of the vertical flapping velocity is determined by the direction of the flapping motion: a minus sign for upward blade flapping, a plus sign for downward flapping. Since the maximal upward blade flapping velocity occurs at the 90° azimuth, the negative angle of attack increment will be greatest at this azimuth and the angle of attack of a given blade element will be minimal. The highest downward vertical flapping velocity occurs at the 270° /59 azimuth, and the positive angle of attack increment $\Delta\alpha$ will be maximal at this azimuth. This means that a given blade element has its maximal angle of attack at the 270° azimuth (Figure 42). Moreover, in analyzing the curve we see that the maximal magnitude of the negative angle of attack increment at the 90° azimuth is less than the maximal magnitude of the positive angle of attack increment at the 270° azimuth.

This variation of the angle of attack increment is explained by the fact that in (17a) for $\psi = 90°$ the second term of the denominator is positive, and $\mathrm{tg}\ \Delta\alpha$ will decrease as a result of increase of the resultant flow velocity over the blade.

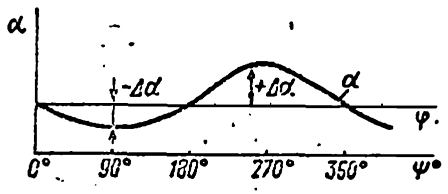

Figure 42. Azimuthal variation of blade element angle of attack.

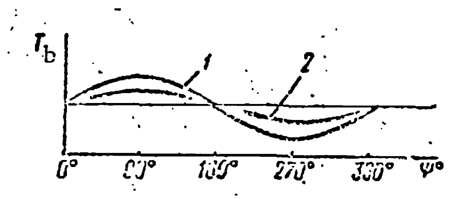

Figure 43. Azimuthal variation of blade thrust:

1) with rigid mounting;
2) with hinged support.

For ψ = 270° the second term of the denominator is negative, and this means that the angle of attack increment $\Delta\alpha$ will increase as a result of reduction of the resultant velocity of the blade element. Moreover, at the 90° azimuth the vertical upward flapping velocity V_{f1} will be less than at the 270° azimuth, when the blade flaps downward. But the blade element angle of attack does not change only in azimuth. It also varies along the main rotor radius (Figure 43). We see from the figure that the angles of attack will be highest for the tip elements at an azimuth close to 270°, and lowest at the 90° azimuth, with the angles of attack being nearly the same for elements at different radii.

The following azimuthal variation of the angle of attack is characteristic: from the 0° azimuth the angles of attack, remaining nearly constant along the length of the blade, decrease up to about the 110° azimuth and then begin to increase.

The following angle of attack variation along the radius is characteristic of the retreating blade: from the root to the tip of the blade the blade element angles of attack increase by 4-5°, with the variation being less at the root elements than at the tip. The angle of attack variation equalizes the blade thrust force azimuthally (Figure 44), and the blade flapping motions are reduced.

§ 32. Effect of Number of Blades on Main Rotor Aerodynamic Characteristics

Single-blade main rotors are not used because of the high degree of unbalance. /60

The primary advantage of the two-blade main rotors is the simplicity of the construction. But the two-blade rotor has low solidity and consequently poor aerodynamic characteristics (low thrust coefficient C_T).

Increase of the solidity with increase of the area of each blade (by increasing its width) leads to increase of the profile drag and reduction of the main rotor efficiency.

Moreover, the blades of any rotor cannot be made perfectly identical. They always differ from one another in their characteristics; therefore the overall blade thrust varies in the forward flight regime. The main rotor resistance to rotation will also vary, i.e., the load on the rotor shaft will vary, and torsional vibrations of the shaft, main rotor vibrations, and vibrations of the entire helicopter will develop.

These problems can be resolved by increasing the number of blades. The larger the number of blades, the smaller the amplitude of the main rotor thrust variations and the smaller the azimuthal variation of the rotor torque, i.e., the rotor becomes more balanced. However, at the same time rotor fabrication and blade balancing and adjustment become more difficult. On this basis, main rotors with 4-5 blades are most frequently encountered.

§ 33. Azimuthal Variation of Rotational Resistance Forces and Reactive Torque

Both the rotational resistance forces and the thrust forces of the blades vary azimuthally as a function of the resultant flow velocity over the blades.

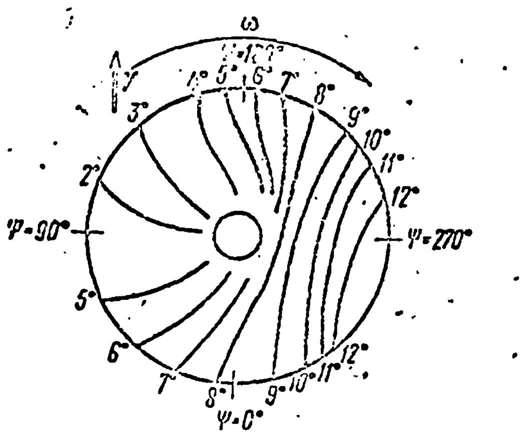

Figure 44. Blade element angle of attack diagram.

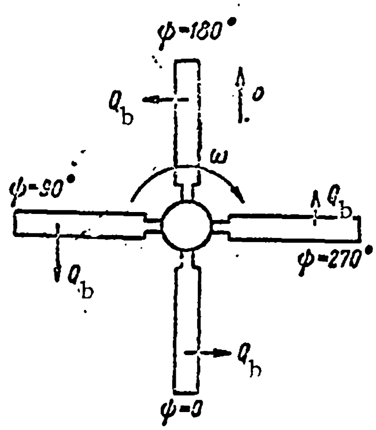

Figure 45. Blade rotational resistance forces at different azimuths.

The resistance to rotation of a single blade reaches its maximal value at the 90° azimuth and minimal value at the 270° azimuth.

At the 0° and 180° azimuths the rotational drag in the forward flight regime is equal to the drag in the axial flow regime, if the main rotor pitch and flight altitude remain the same (Figure 45).

As a result of this variation of the resistance to rotation, there will be azimuthal variation of the main rotor reactive torque from the maximal value when the blades are at the 90° and 270° azimuths to the minimal value when they are located at the 0° and 180° azimuths. /61

The variation of the reactive torque causes vibration (torsional oscillations) with a frequency equal to the main rotor rpm or some multiple thereof. For two blades at opposite azimuth angles the rotational resistance forces are directed oppositely relative to the rotor diameter. At the 0° and 180° azimuths their sum is zero; however, at the 90° and 270° azimuths the sum of these forces is not equal to zero and is directed opposite the helicopter

flight direction, since Q_b is larger at the 90° azimuth than at the $\psi = 270°$ azimuth. This force is the profile drag of the main rotor.

§ 34. Inertial Forces Acting on Main Rotor Blades

As a result of rotation of the main rotor, centrifugal forces, whose magnitude we have already determined, act on the blades.

As a result of the flapping motions, inertial forces develop in the plane perpendicular to the main rotor plane of rotation. The flapping motion inertial forces change their direction and magnitude as a function of blade azimuth.

At azimuths from 270° to 90° the inertial forces of the flapping motions are directed downard. These forces reach their maximal magnitude at an azimuth close to 360°, since the maximal upward acceleration of the blade occurs at this point. At azimuths from 90° to 270° the inertial forces are directed upward and have their maximal magnitude at the 180° azimuth, when the blade acceleration downward will be greatest. At the 90° and 270° azimuths the flapping motion inertial forces are zero, since the flapping motion accelerations are zero at these azimuths, and the flapping motion velocities are maximal.

The inertia forces increase the loads on the main rotor blades.

Blade Coriolis forces. In addition to the centrifugal forces and the flapping motion inertial forces, there are the rotational inertia forces, or Coriolis forces. They arise as a result of combination of the circular blade motion and blade motion relative to the horizontal hinge axis (flapping motion). As a result of variation of the flapping angle during flapping motions, there is a change of the radius of the circle along which the blade center of gravity travels. Thus, Figure 46a shows that with increase of the flapping angle from β_1 to β_2 the radius of the circle described by the blade center of gravity decreases from r_1 to r_2. Therefore, the flapping motions

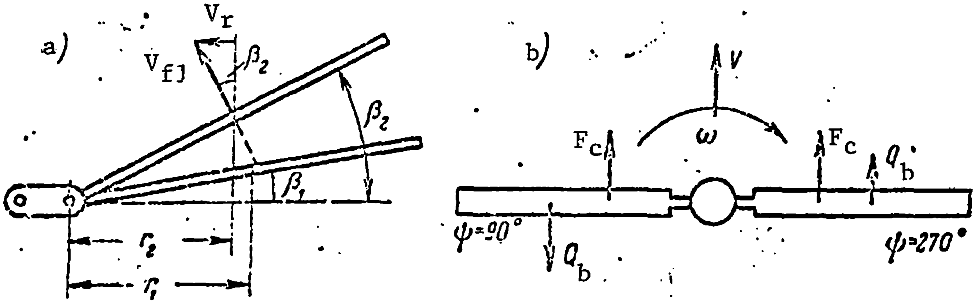

Figure 46. Blade Coriolis forces.

are associated with radial displacement of the blade mass, and this leads to the development of an inertial force which is termed the Coriolis or rotational force.

The essence of the Coriolis force is easily explained if we recall the nature of inertial forces in the case of rectilinear acceleration of motion. Everyone knows from his own experience that during braking the inertia force is directed forward, and that during rectilinear acceleration it is directed aft. Let us apply this rule to the moving blade. /62

When upward flapping takes place, the radius of the blade center of gravity decreases, and the circumferential velocity decreases, i.e., retardation of the motion takes place, and an inertial force appears which is directed forward in the direction of rotation of the main rotor.

During downward flapping the radius of the circle along which the blade center of gravity travels increases, the circumferential velocity $u = \omega r$ increases, and an inertial force directed aft — opposite the rotor rotation — appears.

This analysis is confirmed by the energy conservation law and the associated angular momentum conservation law

$$m_b ur = \text{const}$$

where m_b is the mass of the body;
u is the circumferential velocity;
r is the trajectory radius of curvature.

Let us apply this law to blade motion during variation of the flapping angle.

Since the power supplied to the main rotor shaft remains constant, the angular momentum of each blade must also remain constant

$$m_b u_1 r_1 = m_b u_2 r_2 = \text{const}$$

or

$$m_b \omega_1 r_1^2 = m_b \omega_2 r_2^2 = \text{const}.$$

After dividing through by m, we obtain $\omega_1 r_1^2 = \omega_2 r_2^2 = \text{const}$. We see from this equation that during upward blade flapping $(r_2 < r_1)$ the angular velocity must increase $(\omega_2 > \omega_1)$, in order that the angular momentum conservation law not /63 be violated. A force directed along the main rotor rotation is required in order to increase the angular velocity. This force will be the Coriolis or rotational force.

The Coriolis force F_C is defined as the product of the mass of the body by the acceleration

$$F_C = m_b j_C$$

where m_b is the blade mass;
j_C is the Coriolis acceleration.

The Coriolis acceleration is found from the formula

$$j_C = 2\omega V_r$$

where V_r is the relative or radial velocity of the blade center of gravity.

The velocity V_r (Figure 46a) can be defined as follows:

$$V_r = V_{fl} \sin \beta.$$

Substituting the value of V_r into the formula for the Coriolis acceleration, we obtain

$$j_C = 2\omega V_{fl} \sin \beta.$$

The formula for the Coriolis force finally takes the form

$$F_C = 2 \frac{G_b}{g} \omega V_{fl} \sin \beta.$$

Thus, the blade Coriolis force is directly proportional to blade weight, main rotor rpm, angular flapping velocity, and the flapping angle.

The Coriolis force for the advancing blade is directed in the direction of rotor rotation and increases as the blade approaches the 90° azimuth. Then it begins to diminish and becomes zero at the moment of equilibrium of the blade relative to the horizontal hinge.

The Coriolis force for the retreating blade will be directed aft, opposing rotor rotation, and reaches its maximal value at the 270° azimuth.

The blade Coriolis force develops the moment $M_{Cor} = F_C r_{c.g}$ about the main rotor axis (Figure 46b). For the main rotor with diameter D = 20 m without vertical hinges $M_{Cor} \approx 10{,}000$ kgf·m.

Necessity for vertical hinges. We have established that the rotational drag and Coriolis forces act on the blades in the main rotor rotation plane. At the 90° azimuth these forces are directed in opposite directions (see Figure 46b). At the 270° azimuth these forces coincide in direction. While the moment of the Coriolis force alone reaches a magnitude of about 10,000 kgf·m, the combined moment of the two forces (Coriolis and rotational drag) will be considerably larger. This means that the blade root experiences large /64 loads in the rotor plane of rotation, which can cause rapid failure of the blade if we consider that these loads alter their sign twice per revolution, and the magnitude varies from the minimal to the maximal value twice per revolution.

We encountered the loads created by the blade thrust moment previously. These loads were eliminated with the aid of the horizontal hinge. In order to eliminate the bending moment in the hub rotation plane from the blade root, we must install a vertical hinge. When this hinge is used, the bending moment at the blade root will be zero, i.e., the blade will rotate forward (in the direction of rotor rotation) or aft about this hinge, performing oscillatory motions.

§ 35. Oscillatory Blade Motions

The vertical hinges have stops to limit the oscillatory motions of the blade. However, the blade does not reach the stop in flight, since equilibrium is established under the influence of the moments of the forces acting on the blade in the main rotor hub rotation plane (Figure 47a).

The condition for equilibrium relative to the vertical hinge in general form is expressed by the equality

$$\sum M_{V.H} = 0.$$

For a positive lag angle, this equality can be written as

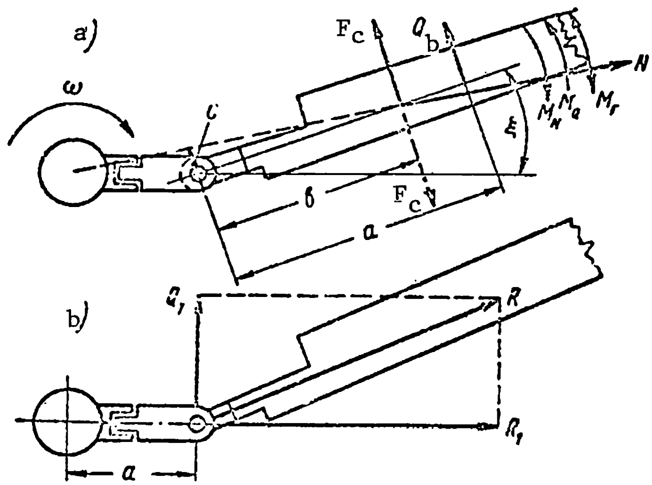

Figure 47. Blade equilibrium about vertical hinge.

$$M_N = M_Q \pm M_F. \tag{18}$$

The lag angle is the angle ξ between the radial line and the longitudinal axis of the blade. The radial line is the line passing through the main rotor axis and the vertical hinge axis. /65

The lag angle will be positive when the blade rotates aft relative to the radial line, opposite the main rotor rotation. In the last equality, the blade centrifugal force moment M_N = Nc will be larger, the larger the centrifugal force and the larger the lag angle. With increase of the lag angle there is an increase of the centrifugal force arm c and its moment relative to the vertical hinge.

For a positive lag angle, the moment M_N rotates the blade ahead in the direction of rotation of the main rotor about the vertical hinge.

If the lag angle is negative, the centrifugal force moment rotates the blade aft, opposite the direction of rotation of the main rotor. Therefore, the centrifugal force moment rotates the blade toward the radial line: it acts as a sort of regulator of the oscillatory motions. Under the influence of this moment, the positive lag angles ξ do not exceed 3-5° (with the main rotor driven by the engine). Negative blade lag angles are developed when the main rotor operates in the autorotation regime. In this case, the lag angles reach 8-12°.

The moment $M_Q = Qa$ of the rotational drag force always opposes rotation of the rotor. Since the force Q_b varies with azimuth, its moment will also vary.

The Coriolis force moment $M_F = F_k b$ varies as a function of azimuth, both in magnitude and direction. At azimuth angles close to 90° the Coriolis force reduces the lag angle, while at azimuths close to 270° the lag is increased.

Now (18) can be written in expanded form

$$Nc = Q_b a \pm F_C b = 0.$$

This will then be the condition for equilibrium of the blade relative to the vertical hinge.

The moments M_Q and M_F vary continuously in azimuth, and their variation is one of the reasons for the oscillatory motions of the blade relative to the vertical hinge in the forward flight regime.

Another reason for the oscillatory motions is the action of the centrifugal force and its moment relative to the vertical hinge. Its action can be compared with the action of the weight force on a freely suspended body.

If a freely suspended body is deflected, oscillations similar to those of a pendulum develop.

Since the centrifugal force is several times stronger than the weight force, it creates significant "pendulous" oscillations, which combine with the oscillations from the variable moments of the rotational drag force and the Coriolis force to amplify or attenuate the amplitudes of the blade /66 oscillations about the vertical hinge.

§ 36. Damping of Oscillatory Blade Motions

If we combine all the forces acting on the blade in the hub rotation plane, we obtain their resultant R. In the case of equilibrium relative to the vertical hinge, the resultant R, shifted to the hinge axis, lies along the blade axis and its moment will be zero. We resolve the force R into two components (Figure 47b): R_1 and Q_1. The force R_1 is radial, and its moment about the hub axis is zero. The force Q_1 creates the moment Q_1a, which twists the rotor shaft. Both the magnitude and moment of the force Q_1 will change with variation of the lag angle. Consequently, the oscillatory motions of the blades about the vertical hinges are the source of torsional vibrations of the shaft, while variation of the force R_1 leads to bending vibrations of the shaft. Various types of dampers are used to eliminate the oscillatory motions (free oscillations) of the blades relative to the vertical hinges. The dampers may be of two types: friction and hydraulic.

The friction dampers consist of a set of steel and cermet (friction) disks (Figure 48). Half of the steel disks are attached to the intermediate link of the hub, the other half is attached to the body of the axial hinge. The friction disks, designed to increase the friction force, are located between the steel disks.

The disks are compressed from above by a spring, which is tightened by a bolt which screws into the finger of the vertical hinge. As the blade rotates about the vertical hinge, friction forces develop between the disks.

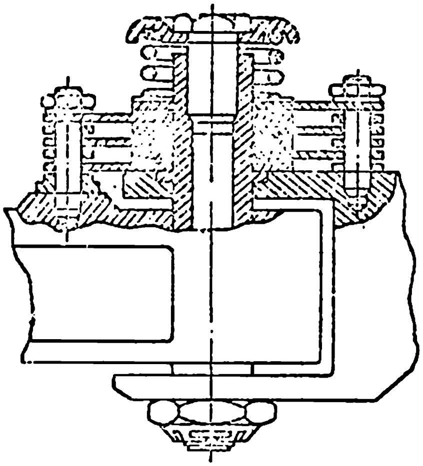

Figure 48. Friction damper for blade vertical hinge.

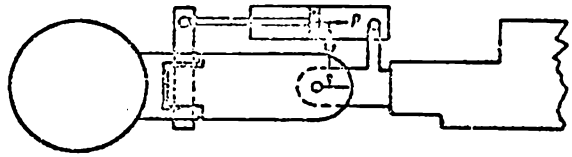

Figure 49. Hydraulic damper for blade vertical hinge.

The moment of these forces about the vertical hinge axis will be the damping moment. The magnitude of the damping moment can be regulated by tightening the bolt. On modern helicopters the damping moment varies from 80 to 120 kgf·m.

The magnitude of the damping moment must be monitored during operations, and care must be taken that it is the same for all the main rotor dampers. /67 With a damper installed, the blade rotates relative to the vertical hinge if the torque exceeds the damping moment. This means that the root portion of the blade experiences a load which does not exceed the magnitude of the damping moment, i.e., the blade root is relieved of a large bending moment. At the same time, the blade will not have free oscillations about the vertical hinge, which means that there will be no reason for the onset of severe vibrations.

The friction dampers can be used on light and intermediate helicopters (Mi-1, Mi-4). They are not used on heavy helicopters because of the small magnitude of the damping moment and the frequent damper regulation required. The hydraulic dampers are being used more and more at the present time.

The hydraulic damper consists of a cylinder and a rod and piston (Figure 49). The cylinder is attached to the body of the vertical hinge, while the rod is attached to the finger of the horizontal hinge. In the piston there are calibrated orifices with relief valves.

As the blade rotates relative to the vertical hinge, the rod and piston displace relative to the cylinder. The cylinder cavities are filled with a liquid. As the piston moves in the cylinder, the liquid opens the relief valves and flows from one cavity into the other through orifices in the piston. The resistance force P is developed. The moment of this force about the vertical hinge axis $M_q = Pa$ will be the damping moment. This moment is easily regulated by selecting the piston area, diameter of the orifices in the piston, and lever arm a (from the damper axis to the vertical hinge axis).

Hydraulic dampers have the following drawbacks:

low damping moments for low rates of blade rotation relative to the vertical hinge;

marked increase of the damping moments during rapid rotation;

dependence of the damping moments on temperature because of variation of the liquid viscosity;

marked variation of the damping moments if air gets into the cylinder chamber.

The hydraulic dampers are sometimes supplemented with spring dampers to eliminate the first problem.

The second problem is eliminated by proper choice of the relief valves. /68

The third problem is alleviated by selection of a liquid whose viscosity depends very little on temperature.

To prevent air entry into the damper, a small supply reservoir is installed on the root portion of the blade, and the damper cavities are replenished with the working fluid from this.

§ 37. Possibility of Loss of Blade Flapping Motion Stability

The main rotor hubs in which the angle between the horizontal hinge axis and the radial line is 90°, i.e., $\delta_{HH} = 90°$, have a serious problem: loss of flapping motion stability. By loss of flapping motion stability, we mean possible deflection of the blade upward or downward to the horizontal hinge stops. This phenomenon takes place as a result of variation of the blade incidence angles during flapping, together with the presence of a blade lag angle.

If the blade rotates relative to the vertical hinge through the lag angle ξ, then the blade element chord AB will not be parallel to the horizontal hinge axis. During flapping motions, the leading edge and trailing edge of the blade element will displace along two different radii: the point A at the leading edge will have the larger radius r_A, the point B on the trailing edge will have the radius r_B (Figure 50).

When the blade flaps upward through a certain flapping angle β, the points A and B move up different distances relative to the main rotor hub rotation plane. Point A will move to the height h_A, while point B moves to the height h_B. As a result of this height difference, the additional angle $\Delta\alpha$ develops /69
between the blade element chord and the hub rotation plane. The larger the blade lag angle and the larger the change of the flapping angle, the larger the incidence angle increase will be.

We see from the figure that

$$\sin \Delta\varphi = \frac{\Delta h}{b};$$

where b is the blade element chord length.

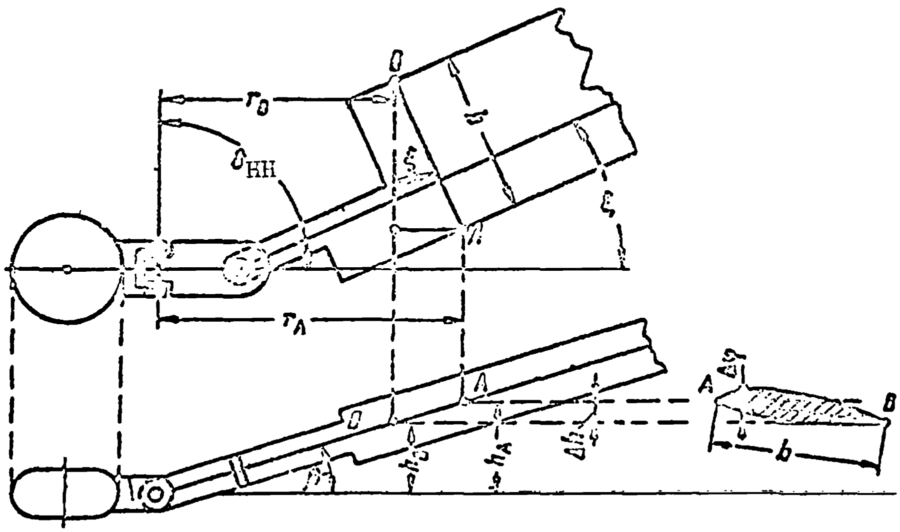

Figure 50. Blade pitch change during flapping motions.

The increased height of the leading edge above the trailing edge Δh is found from the formula

$$\Delta h = a \operatorname{tg} \Delta\beta,$$

where $a = b \sin \xi$ is the projection of the chord on the radial line.

Consequently

$$\sin \Delta\varphi = \frac{\Delta h}{b} = \frac{a \operatorname{tg} \Delta\beta}{b} \doteq \frac{b \sin \xi \operatorname{tg} \Delta\beta}{b},$$

or

$$\sin \Delta\varphi \doteq \sin \xi \operatorname{tg} \Delta\beta,$$

i.e., the previously drawn conclusion is confirmed.

If there is a lag angle ξ, the blade element incidence angle will increase during up-flapping and will decrease during down-flapping. This variation leads to increase of the blade thrust and its moment relative to the horizontal hinge during upward flapping, i.e., the blade will travel up against the stop.

During down-flapping of the blade, blade thrust will decrease still further, which leads to downward travel of the blade against the lower stop. This is then the manifestation of the loss of flapping motion stability.

How can these undesirable phenomena be eliminated? The simplest technique is to increase the degree of pitch horn compensation, i.e., increase the compensation coefficient. However, increase of this coefficient leads to an increase of a particular type of main rotor blade vibration — a vibration of the flutter type. Therefore, the loss of flapping motion stability is eliminated at the present time by a different approach — a change of the basic geometry of the main rotor hub.

To accomplish this, a hub is used in which the angle between the horizontal hinge axis and the longitudinal blade axis with the blade in the radial position is less than 90°, i.e., $\delta_{HH} < 90°$ (Figure 51). Such a hub is installed, for example, on the Mi-1 helicopter.

If the blade of such a hub is rotated through the lag angle $\xi = 90° - \delta_{HH}$, its longitudinal axis is then perpendicular to the horizontal hinge axis. This means that the radii of rotation about the horizontal hinge for the /70
leading and trailing edges approach one another, i.e., $r_A = r_B$ (see Figure 50). In this case, flapping motions will not lead to any height increment Δh. Therefore, there will not be any increase of the incidence angle Δh, and the flapping motions remain stable.

If the lag angle $\xi > 90° - \delta_{HH}$, the incidence angles vary just as for $\delta_{HH} = 90°$. However, in this case, the instability of the flapping motions

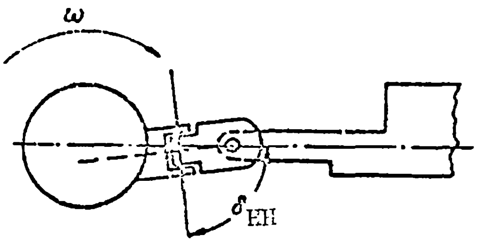

Figure 51. Schematic of main rotor hub.

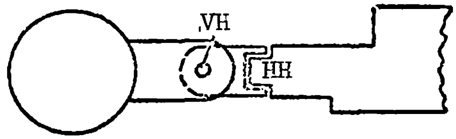

Figure 52. Schematic of main rotor hub.

shows up to a lesser degree.

However, if the vertical and horizontal hinges of the main rotor are located in the reverse order, the factors which cause loss of flapping motion stability can be completely eliminated (Figure 52).

With this hinge arrangement, rotation of the blade about the vertical hinge does not cause any change of the position of the blade element chord relative to the horizontal hinge axis.

Consequently, for this hub the radii of rotation of the leading edge and trailing edges about the horizontal hinge axis will always be the same, ($r_A = r_B$), i.e., there will not be any change of pitch during flapping motions, and loss of flapping motion stability will not occur.

Programmed Testing Questions and Answers

Question 1. Main rotor thrust dependence on flight velocity in the forward flight regime.

Answer 1. Main rotor thrust in the forward flight regime is defined by the formula

$$T = 2\rho F V_1 V_i.$$

Since $V_1 > V_i$, the thrust increases with increase of the flight speed. But with increase of the flight speed, there is at the same time a reduction of the induced velocity. Therefore, the product $V_1 V_i$ increases boundedly and reaches a maximal value at a speed of about 100 km/hr. Thus, the main rotor thrust is maximal at this flight speed.

Answer 2. The main rotor thrust in the forward flight regime is defined by the formula

$$T = C_T \frac{\rho}{2} F W^2 .$$

Since the resultant velocity W = u + V, the main rotor thrust increases /71
with the flight speed.

Answer 3. The main rotor thrust in the forward flight regime is defined by the formula

$$T = 2\rho F V_i^2.$$

With increase of the flight speed, the induced velocity V_i increases therefore, the thrust developed by the rotor will increase.

Question 2. What is the resultant velocity of the blade element in the forward flight regime?

Answer 1. The velocity equal to the vector sum of the flight speed and the induced inflow velocity $V_1 = V + V_i$. With increase of the flight speed the resultant velocity increases, and this leads to increase of the main rotor thrust.

Answer 2. The velocity equal to the vector sum of the circumferential velocity of the blade element and the induced inflow velocity

$$\bar{W} = \bar{u} + \bar{V}_i.$$

The angle between the resultant velocity of the blade element and the chord is the angle of attack.

Answer 3. The velocity equal to the vector sum of the circumferential velocity of the blade element and the projection of the flight velocity vector on the line of the circumferential velocity vector

$$\bar{W} = \bar{u} + \bar{V} \sin \psi.$$

The resultant velocity varies azimuthally. At the azimuths $\psi = 0°$ and $\psi = 180°$ $W = u$; at the azimuth $\psi = 90°$ $W = u + V$; at the azimuth $\psi = 270°$ $W = u - V$.

Answer 4. The resultant velocity is the projection of the helicopter flight velocity on the main rotor hub rotation plane ($\bar{W} = \bar{V} \cos A$). It depends on the flight speed and the main rotor angle of attack.

Question 3. What does a change of the resultant velocity of the blade element in the forward flight regime lead to?

Answer 1. Change of the resultant velocity in the forward flight regime leads to change of the main rotor angle of attack, increase of the thrust, appearance of a rolling moment of the main rotor, and nonuniform loading on the blades.

Answer 2. Change of the resultant velocity in the forward flight regime leads to azimuthal variation of the blade thrust. The consequence of the azimuthal variation of the blade thrust for a main rotor with rigid blade

attachment will be the appearance of a rolling moment and vibration; for the main rotor with hinged blade attachment, flapping motions will appear.

Answer 3. Azimuthal change of the resultant velocity leads to the appearance of a rolling moment of the main rotor, change of the angle of attack, flow separation from the root region of the advancing blade, and limitation of the helicopter flight speed.

Question 4. Reasons for the occurrence of and consequences of blade flapping motions in the forward flight regime.

Answer 1. The reason for the occurrence of flapping motions in the forward flight regime is the disruption of the condition of blade equilibrium relative to the horizontal hinge as the blade thrust varies with azimuth. As a result of the flapping motions, there is a change of the blade flapping angle in azimuth, which leads to tilting of the main rotor coning axis and change of the thrust vector direction. Moreover, as a result of the flapping motions the blade element angles of attack change and Coriolis forces develop.

Answer 2. The reason for the blade flapping motions in the forward flight regime is the azimuthal variation of the blade pitch, which results in change of the blade thrust and the appearance of the flapping motions. The result of the flapping motions is tilt of the main rotor coning axis forward /72 and in the direction of the advancing blade, and increase of the helicopter flight speed.

Answer 3. The reason for the occurrence of blade flapping motions in the forward flight regime is the azimuthal variation of the blade element angles of attack. As a result, there is a change of the blade thrust, which then leads to the flapping motion. The consequence of the flapping motions is a tilt of the coning axis and the appearance of main rotor thrust components directed aft and toward the retreating blade.

Question 5. Blade flapping compensator and its purpose.

Answer 1. The blade flapping compensator consists of the connection of the cyclic control rod with the pitch control horn, which controls the variation of the pitch angle ϕ. The objective is to limit the blade flapping motions. The flapping compensator effectiveness will be the higher, the greater the distance between the blade longitudinal axis and the pitch horn hinge.

Answer 2. The blade flapping compensator consists of the connection of the lever with the aid of which the blade pitch is changed with the rod coming from the cyclic control. During upward flapping the pitch is reduced, during down flapping the pitch is increased. The flapping compensator effectiveness will be the higher, the smaller the distance from the horizontal hinge axis to the pitch horn hinge.

Answer 3. The blade flapping compensator involves the particular positioning of the pitch horn hinge. This position is defined by the distance a from the horizontal hinge axis and by the distance b from the blade longitudinal axis. The larger the ratio a/b, the more effective the flapping compensator. The compensator is designed to limit blade flapping motions, which is achieved by reducing blade pitch during up-flapping and increase of the pitch during down-flapping.

Question 6. Reasons for and consequence of azimuthal variation of the blade element angle of attack in the forward flight regime.

Answer 1. The blade element angles of attack vary in azimuth as a result of pitch change. The angle of attack of the advancing blade increases, while that of the retreating blade decreases. With change of the angle of attack there is a change of the blade thrust, which leads to azimuthal equalization of the thrust force.

Answer 2. In the forward flight regime the vertical flapping velocity of the advancing blade is directed upward, therefore, the angle of attack of each blade element decreases while that of the retreating blade increases. The angle of attack variation leads to azimuthal equalization of the blade thrust force.

Answer 3. The blade element angles of attack change azimuthally as a result of the flapping motions. During the flapping motions the induced flow velocity changes, and therefore an angle-of-attack increment $\Delta\alpha = \frac{V_l}{u + V \sin\psi}$ appears. The angles of attack of the advancing blade decrease, while those of the retreating blade increase, and at the 270° azimuth the angle of attack may become greater than the critical value, and flow separation occurs.

Question 7. Blade Coriolis force and its azimuthal variation.

Answer 1. The blade Coriolis force is the force which develops as a result of the combination of two velocities: the circumferential velocity of the blade center of gravity and the radial velocity which develops as a result of variation of the flapping angle.

The Coriolis force of the advancing blade is directed in the direction of rotation of the motor and reaches its maximal value at the 90° azimuth. The Coriolis force of the retreating blade is directed opposite the rotor rotation and reaches its maximal value at the 270° azimuth.

Answer 2. The Coriolis force is an inertial force which arises in the forward flight regime as a result of the combination of the circumferential velocity of the blade center of gravity and the helicopter translational flight velocity. For the advancing blade, this force is directed forward and reaches its maximal magnitude at the 90° azimuth; for the retreating blade it is directed aft and reaches its maximal value at the 270° azimuth.

Answer 3. The Coriolis force is an inertial force which arises from flapping motions resulting from the combination of the angular velocity of motion of the blade center of gravity and the vertical flapping velocity. For the advancing blade, this force is directed upward and reaches its maximal value at the 90° azimuth; for the retreating blade, it is directed downward and will be maximal at the 270° azimuth. /73

CHAPTER V

HELICOPTER VERTICAL FLIGHT OPERATING REGIMES /73

Concept of helicopter flight regimes. The helicopter flight regime can be either steady state or nonsteady state. Rectilinear flight at constant velocity is termed a steady state regime. The steady-state regimes can be subdivided as follows.

1. Vertical flight regimes:
   hovering;
   vertical climb;
   vertical descent.

There are two varieties of vertical descent: descent with engine operating and descent in the main rotor autorotation regime.

2. Horizontal flight regime.

3. Climb regime along inclined trajectory.

4. Descent regime along inclined trajectory (can be performed with engine operating or with main rotor autorotating).

The unsteady flight regime is one in which the velocity vector changes in magnitude or direction. The unsteady regimes include takeoff, landing, maneuvering (horizontal turns, heading changes, spiral, snaking, and so on) and transition from one flight regime to another.

In accordance with the law of inertia, a body travels uniformly and rectilinearly or is in a state of rest if no external forces act on it. The steady-state flight regime is uniform, rectilinear motion of the helicopter. Consequently, for the realization of this regime it is necessary that the geometric sum of the forces acting on the helicopter in flight be equal to zero. Moreover, the sum of the force moments acting on the helicopter relative to the center of gravity must also be equal to zero. These will then be the conditions for complete equilibrium of the helicopter.

Unsteady flight can occur only if some unbalanced force acts on the helicopter and imparts an acceleration to it, i.e.,

$$\Sigma F_{cg} \neq 0 \text{ and } \Sigma M_{cg} \neq 0.$$

§ 38. Hovering Regime. General Characteristics /74

The helicopter hovering regime is that flight regime in which the velocity equals zero. Hovering can be performed relative to the air and relative to the ground. If the air is stationary relative to the earth, i.e., the wind velocity equals zero ($u = 0$), the helicopter hovering relative to the air will be at the same time hovering relative to the Earth.

If the wind velocity is greater than zero, when hovering relative to the Earth (when the nose of the helicopter is pointed into the wind), it will perform flight relative to the air with the velocity of the wind. In this case the main rotor will operate in the forward flight regime. When the helicopter hovers relative to the air, the main rotor operates in the axial flow regime.

If during hovering relative to the air, there is a wind and the helicopter's nose is pointed into the wind, the helicopter will move backward with the velocity of the wind.

If in the presence of a wind the helicopter plane of symmetry is at an angle of 90° to the wind direction, the helicopter will displace to the side relative to the Earth (when hovering relative to the air) or relative to the air (when hovering relative to the Earth).

Hovering is performed in every flight during takeoff and landing. In addition, hovering is performed during unloading and loading when it is not possible to land (for example, over water, brush, rough ground, and in other such situations). Therefore, hovering must be performed relative to the Earth. In this case the pilot maintains the helicopter stationary relative to some point on the ground at a height of no more than 10 meters. Hovering at a height of more than 10 m and less than 200 m is hazardous, since in case of engine failure a safe emergency landing is not assured. Hovering at higher altitudes is performed only relative to the air, since the pilot cannot maintain the helicopter stationary relative to the ground from a high altitude. The helicopter speed relative to the air must not be less than that which can be indicated stably by the airspeed indicator meter (40 km/hr).

§ 39. Diagram of Forces Acting on Helicopter and Hovering Conditions

In the further study of the hovering regime we examine helicopter hovering relative to the air with the main rotor operating in the axial flow regime.

In order to avoid complicating our understanding of the question, we shall assume that the wind velocity is zero.

During hovering, it is necessary to observe the general conditions which characterize any steady-state flight regime, i.e., /75

$$\Sigma F_{cg} = 0 \text{ and } \Sigma M_{cg} = 0.$$

The following basic forces act on the helicopter during hovering (Figure 53a):

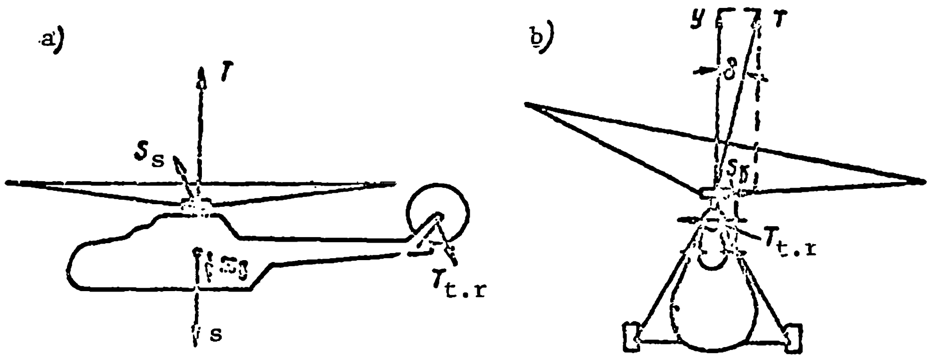

Figure 53. Forces acting on helicopter in hovering regime.

helicopter weight G;

main rotor thrust T;

tail rotor thrust $T_{t.r}$;

parasite drag X_{par}.

The parasite drag force arises as a result of air flow from the main rotor over the fuselage and other parts of the helicopter. This force is very small and amounts to about 1-2% of the helicopter weight. The main rotor thrust increases somewhat as a result of the air flow over the fuselage, which means that the effect of the force X_{par} decreases, and it can hereafter be neglected.

The main rotor reactive moment during hovering is balanced by the tail rotor thrust moment $M_r = T_{t.r}L_{t.r}$. This is necessary to prevent the helicopter from turning about the vertical axis. But in this case the unbalanced force $T_{t.r}$ acts on the helicopter and the helicopter displaces to the side. To prevent lateral displacement it is necessary to balance the tail rotor thrust by a force directed oppositely. To this end the main rotor thrust vector is deflected to the side opposite the direction of the tail rotor thrust. For helicopters with right hand rotation of the main rotor (as seen from above) the tail rotor thrust is directed to the left (Figure 53b). As a result of tilting of the cone axis to the right through the angle δ, the main rotor side thrust develops

$$S_s = T\sin\delta,$$

which balances the tail rotor thrust. The vertical component $Y = T\cos\delta$ of the main rotor thrust will be balanced by the helicopter weight.

The angle δ does not exceed 3-5°. And since $\cos 5° \approx 1$, we can say with adequate precision that $Y \approx T$. Thus, the helicopter hovering conditions are expressed by the equalities /76

$$Y = G \text{ or } Y - G = 0,$$

$$T_{t.r} = S_s \text{ or } T_{t.r} - S_s = 0. \ \Sigma M_{cg} = 0.$$

Since there are no forces acting along the helicopter longitudinal axis in the hovering regime, $\Sigma M_{cg} = 0$ is assured.

In view of the equality $Y \approx T$ we can write the first hovering regime condition in the form T = G. We shall use this equality hereafter. Therefore, for helicopter hovering it is necessary that:

$T = G$ (constant hovering height);

$T_{t.r} = S_s$ (absence of lateral displacement);

$\Sigma M_{cg} = 0$ (absence of rotation about the center of gravity).

The hovering regime is a characteristic flight regime and defines to a considerable degree the helicopter's flight characteristics.

<u>Example</u>. A helicopter is hovering. $G = 2200$ kgf, $N_e = 575$ hp, $\zeta = 0.78$, $n = 249$ rpm, $L_{t.r} = 8.65$ m. Find: N, M_r, $T_{t.r}$, and δ.

<u>Solution</u>. 1. The power expended in rotating the main rotor is defined with account for the power utilization coefficient

$$N = N_e \xi = 575 \times 0.78 = 450 \text{ hp}.$$

2. From (11) we find the main rotor reactive moment $M_r = \frac{N_r}{\omega}$.

The main rotor angular velocity is found from the formula

$$\omega = \frac{2\pi n}{60} = \frac{\pi n}{30} = \frac{3.14 \cdot 249}{30} = 26 \text{ rad/sec}$$

$$M_r = \frac{N_n 75}{\omega} = \frac{450 \cdot 75}{26} = 1295 \text{ kgf.m}$$

3. The tail rotor thrust is found from the equality

$$T_{t.r} = \frac{M_r}{L_{t.r}} = \frac{1295}{8.65} = 150 \text{ kgf}.$$

4. The tilt of the thrust vector is found from the equality

$$\sin \delta = \frac{S_s}{T} = \frac{T_{t.r}}{G} = \frac{150}{2200} \approx 0.068;$$

$$\arcsin 0.068 = 4°.$$

§ 40. Thrust and Power Required for Hovering

The thrust required for helicopter hovering is found from the Formula $T = G$. If we take into account the rotor slipstream flowing over the fuselage, the thrust increases by 1-2% in comparison with the weight. But how can we obtain main rotor thrust equal to the weight of the helicopter? Let us turn /77
to the Formula $T = C_T F \frac{\rho}{2} (\omega R)^2$. The main rotor thrust depends on the thrust coefficient G_T, air density ρ, and rotor angular velocity or rpm. Knowing the main rotor rpm and the air density in the standard atmosphere, we can calculate the thrust. The thrust coefficient C_T can be found from the formulas

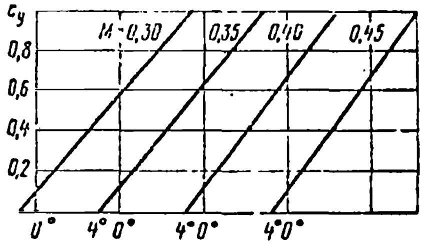

Figure 54. Blade element lift coefficient versus angle of attack (Mi-1).

$$C_T = \frac{C_{y_7}\sigma_7}{3.2},$$

$$C_T \approx \frac{\alpha_7^0 \sigma_7}{32}, \qquad (19)$$

where C_{y_7} is the lift coefficient of the blade element with relative radius $\bar{r} = 0.7$;

σ_7 is the solidity ratio at the radius $\bar{r} = 0.7$;

α_7^0 is the angle of attack of the blade element located at the relative radius $\bar{r} = 0.7$.

Figure 54 shows the blade profile lift coefficient as a function of angle of attack for various M.

We find C_y from the figure and then compute both C_T and main rotor thrust at full rpm at the altitude H = 0 for maximal pitch. With account for blade twist $\varphi_7 = 9°$, the induced downwash angle β for the given blade element equals about 4° (see Figure 15b). Then the blade element angle of attack is

$$\alpha_7 = \varphi_7 - \beta = 9 - 4 = 5°.$$

From the same figure we find $C_{y_7} = 0.42$. Knowing that the solidity σ for the Mi-1 main rotor is constant along the radius and equal to 0.05, we find

$$C_T = \frac{C_{y_7}\sigma_7}{3.2} = \frac{0.42 \cdot 0.05}{3.2} = 0.0066.$$

We then find the thrust at maximal engine rpm, knowing that $F = 162\ m^2$ and $\omega = 26$;

$$T = C_T F \frac{\rho}{2} (\omega R)^2 = 0.0066 \cdot 162 \frac{0.125}{2} (26 \cdot 7.17)^2 = 2250 \text{ kgf}.$$

This force equals the maximal flight weight for which the Mi-1 can make a vertical takeoff.

We can also determine the coefficient C_T from (19), but with less accuracy

$$C_T = \frac{a_7^0 \sigma_7}{32} = \frac{5 \cdot 0.05}{32} = 0.007.$$

Hovering can be performed at different altitudes relative to sea level /78
and at different air temperatures; therefore, we shall examine the dependence of main rotor thrust on the air density.

Air density varies with change of temperature and pressure and, therefore, depends on the altitude, time of day, and season of the year.

If in the Formula $T = C_T F (\omega R)^2 \frac{1}{2} \rho$ the product $C_T F (\omega R)^2 \frac{1}{2} = \text{const}$, then for $H=0$ $\rho = \rho_0$; for $H>0$, $\rho = \rho_H$; $T_0 = \text{const}\, \rho_0$; $T_H = \text{const}\, \rho_H$.

Let us divide the first equality termwise by the second:

$$\frac{T_0}{T_H} = \frac{\rho_0}{\rho_H}, \text{ hence } T_H = T_0 \frac{\rho_H}{\rho_0}. \tag{20}$$

The ratio $\frac{\rho_H}{\rho_0} = \Delta$ is called the relative density.

This means that,

$$T_H = T_0 \Delta. \tag{21}$$

This formula shows that the thrust developed by the rotor will diminish with increase of altitude and temperature. In order to accomplish hovering during

takeoff from a high-altitude airfield or at high air temperatures, it is necessary to increase the thrust to the magnitude of the weight force by increasing rotor pitch and rpm. But (20) and (21) indicate that helicopter takeoff weight decreases for takeoff from a high-altitude airfield or with increased air temperature. Therefore, the helicopter has lower takeoff weight in the summer than in the winter.

The power required for helicopter hovering must be supplied to the main rotor shaft in order to overcome the retarding action of the reactive moment. It is known that $N_{req} = M_{r\omega}$. But at constant rpm

$$M_r = M_{tor} = m_{tor} F \frac{\rho}{2} (\omega R)^2 R.$$

Therefore, the required main rotor torque and power at constant rpm at constant altitude depend on the torque coefficient m_{tor}. It was shown in Chapter 3 that $m_{tor} = m_{tor_i} + m_{tor_{pr}}$, i.e., this torque coefficient is made up from the induced drag coefficient and the profile drag coefficient. The induced drag coefficient depends on the induced velocity.

The coefficient m_{tor} and the relative induced velocity $\bar{V}_i$ are defined by the respective formulas:

$$m_{tor} = 1.08 C_T \bar{V}_{i_7} + \frac{\sigma_7}{400}; \quad \bar{V}_i = \frac{V_i}{\omega R}, \tag{22}$$

where $\bar{V}_{i_7}$ is the relative induced velocity of the element with $\bar{r} = 0.7$

$$\bar{V}_{i_7} = 0.52 \sqrt{C_T}.$$

Knowing the value of C_T for the Mi-1 rotor, we can find the relative induced velocity

$$V_{i_{\text{v}}} = 0.52\sqrt{0.0066} = 0.043.$$

We substitute this value into (22) and find m_{tor}

$$m_{tor} = 1{,}08 C_T V_{i_{\text{v}}} + \frac{\sigma_7}{400} = 1.08 \cdot 0.0066 \cdot 0.043 + \frac{0.05}{400} \approx 0.00043.$$

From (11) and (10) we find the power required for hovering the Mi-1 helicopter at maximal engine rpm.

$$N_{req} = M_r \omega = m_{tor} F \frac{\rho}{2} (\omega R)^3 = 0.00043 \cdot 162 \times$$

$$\times \frac{0.125}{2} \cdot 186^3 \approx 28\,000 \text{ kgf.m/sec}$$

or

$$N_{req} = \frac{28\ 000}{75} = 374 \text{ hp}$$

Knowing the power utilization coefficient $(\zeta = 0.78)$, we find the power which must be developed by the engine in hovering

$$N_e = \frac{N_r}{\zeta} = \frac{374}{0.78} \approx 480 \text{ hp}$$

This power is somewhat less than that developed by the AI-26C engine at takeoff power at sea level (575 hp).

The power required to overcome blade induced drag (in creating the induced velocity) can be found from the formula

$$N_i = \frac{TV_i}{75\chi}. \tag{23}$$

Consequently, this power depends on helicopter weight and air density.

The power required to overcome profile drag can be found from the formula

$$N_{pr} = \frac{T\omega R\nu}{100\chi}, \tag{24}$$

where ν is the inverse of the blade element efficiency.

The ratio of the profile drag coefficient to the lift coefficient is termed the reciprocal efficiency of the blade element /80

$$\nu = \frac{C_{x_{pr}}}{C_y}.$$

The blade element reciprocal efficiency varies in the range 0.02–0.04. We see from (24) that the profile power depends on the main rotor rpm. Moreover, if the blade surface roughness is increased, the profile drag coefficient increases and the reciprocal aerodynamic efficiency of the blade element increases. This circumstance must be considered in the wintertime, when the blade may be covered with frost, which leads to a large increase of the profile power.

The profile drag coefficient is highest for blades with fabric covering, lower for blades with plywood covering, and still lower for metal blades. Therefore, main rotors with metal blades are most widely used at the present time. They have recently been installed on the Mi-1 and Mi-4 helicopters and on all the new helicopters.

We can use (23) and (24) to find the induced and profile powers for the Mi-1 helicopter ($G = 2200$ kgf; $V_i = 7.5$ m/sec, $\chi = 0.9$; $\omega = 26$; $\nu = 0.03$)

$$N_i = \frac{TV_i}{75\chi} = \frac{2200\cdot 7.5}{75\cdot 0.9} = 245 \quad \text{hp};$$

$$N_{pr} = \frac{2200\cdot 26\cdot 7.17\cdot 0.03}{100\cdot 0.9} = 136 \quad \text{hp};$$

$$N_{req} = N_i + N_{pr} = 245 + 136 = 381 \text{ hp}.$$

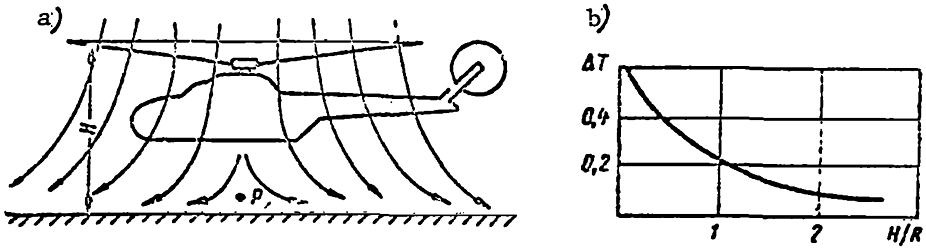

Figure 55. Hovering at low height.

This result nearly coincides with that obtained using (10) and (11) — 374 hp.

In the hovering regime the thrust required equals the helicopter weight. If low power is required to achieve this condition, the main rotor has high relative efficiency. On the average the magnitude of this coefficient varies in the range 0.6–0.65 if the main rotor blades operate at the optimal angles of attack (5–6°).

Effect of air cushion on hovering. The so-called air cushion (Figure 55a) develops during helicopter hovering at low height ($H < D$). The essence of this phenomenon is as follows. The air from the rotor travels downward and its velocity decreases to zero as it encounters the ground. In this case the pressure below the helicopter increases as a result of the velocity head. The total pressure at the center of the disk, projected onto on the ground, is

$$P = P_{at} + \frac{\rho V_i^2}{2},$$

where P_{at} is atmospheric pressure.

The thrust force increases as a result of the pressure increase below the rotor. For $H = 0.2R$ the main rotor thrust increases by 50% in comparison with the thrust without the air cushion effect, for $H = R$ the increase is 25%, for $H = 2R$ it is 10%. At the height $H = 4R$ the effect of the air cushion disappears completely. /81

The variation of the thrust increment under the influence of the air cushion is shown in Figure 55b. The air cushion is used in takeoff with an overload or from a high-altitude airfield, when there is a shortage of power.

The air cushion effect has a favorable influence on helicopter stability, since there will be an increase of the thrust for that portion of the main rotor which is closest to the ground when the rotor tilts to one side, and this leads to the development of a righting moment.

§ 41. Vertical Climb

Helicopter flight along a vertical trajectory with constant velocity is termed the vertical climb regime. The following forces act on a helicopter in vertical climb (see Figure 53): the helicopter weight force G, the main rotor thrust force T, and the tail rotor thrust force $T_{t.r}$.

In order to balance the tail rotor thrust force, the main rotor thrust force vector must be inclined at the angle δ, which results in the creation of the vertical thrust force component Y and the horizontal component S_s.

The steady-state climb conditions will be expressed by the equalities:

$$Y = G + X_{par},$$

$$T_{t.r} = S_s,$$

$$\Sigma M_{cg} = 0.$$

Since the force X_{par} is small and may be neglected and the force $Y \approx T$, the equality $Y = G + X_{par}$ can be replaced by the equality $T = G$. Then the vertical climb conditions will be analogous to the hovering conditions. The condition $T = G$ assures constant helicopter speed in the vertical climb regime. /82

The equality $T_{t.r} = S_s$ assures rectilinear flight.

Power required for vertical climb. The difference between the hovering and vertical climb conditions is that, first, the force X_{par} in the vertical climb is larger than in hovering, since it depends on two velocities: the vertical velocity V_y and the induced velocity V_i.

Second, while in hovering the equality T = G assures a state of relative rest; in climb the same equality must assure constant velocity of the vertical motion.

Consequently, the work per unit time of the thrust force in a vertical climb is different from that in a hover; during climb this work is made up of the work expended on creating the thrust force equal to the weight ($TV_i = GV_i$), and the work expended in creating the vertical velocity (TV_y). During hovering, the work per unit time of the thrust force is expended only in creating the induced flow velocity and is equal to TV_i.

Therefore, while the induced power required for hover is found from the formula

$$N_i = \frac{TV_i}{75\chi},$$

the induced power required for vertical climb is expressed by the formula

$$N_{i_{cl}} = \frac{T(V_i + V_y)}{75\chi}.$$

For a low climb velocity (2-3 m/sec) the induced velocity differs very little from the induced velocity in hovering, i.e., $V_{i_{hov}} \approx V_{i_{cl}}$. But this implies that the induced power in a climb is greater than the hovering power by the magnitude ΔN (the excess power required for climb in comparison with the hovering power required). Bearing in mind that the profile power in climb is practically equal to the profile power in hover, we can express the formula for the power required for vertical climb through the hovering power formula

$$N_{cl} = N_{hov} + \Delta N.$$

Vertical climb is possible only if excess power is available. To transition from hover to climb, the pilot increases the main rotor pitch with the aid of the "collective-throttle" lever; in this process the main rotor rpm remains nearly constant while the thrust increases. The helicopter transitions from hover to vertical climb. The thrust in a vertical climb can be determined from the formula of ideal rotor momentum theory $T = 2\rho F V_1 V_i$. In this case $V_1 = V_i + V_y$. As the vertical velocity increases the induced velocity will /83 decrease. Therefore, the main rotor thrust again decreases to the value the rotor had in hovering prior to increasing the collective pitch. Thus, in transitioning from hover to climb the pilot actually increases the power supplied to the rotor, but the main rotor thrust force remains nearly unchanged. Therefore,

$$T_{cl} \approx T_{hov}, \text{ but } N_{cl} = N_{hov} + \Delta N.$$

Vertical climb velocity. The vertical climb velocity is the height through which the helicopter center of gravity displaces in one second.

For vertical displacement of any body it is necessary to perform work equal to the product of the weight of the body by the height change, i.e., $A = GH$. Work performed per second is power. This means that to perform a climb additional power must be supplied to the main rotor, which is expended in creating the vertical velocity. This power is the excess power $\Delta N = GV_y$. Hence we find

$$V_y = \frac{\Delta N 75}{G}. \tag{25}$$

The vertical velocity depends on the excess power and the helicopter weight. If the helicopter is heavily overloaded, there is sufficient engine

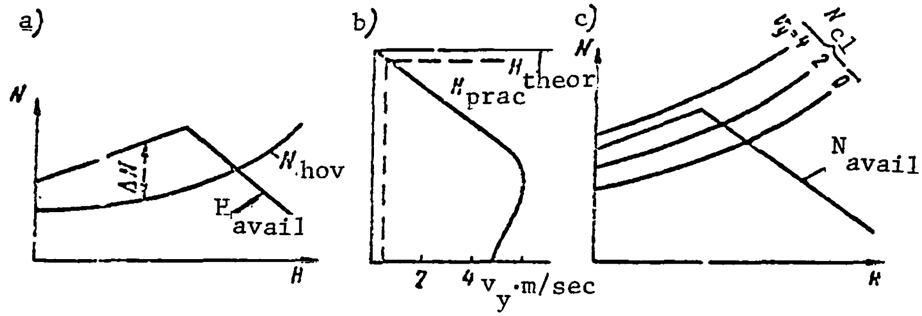

Figure 56. Aerodynamic characteristics in the climb regime.

power only for hovering in the air cushion zone and in this case vertical climb is not possible.

The excess power used for vertical climb is equal to the difference between the power available and the power required for hover

$$\Delta N = N_{avail} - N_{hov}.$$

Usually the helicopter excess power is not large and near sea level does not exceed 10-15% of the total engine power.

The vertical velocity at sea level amounts to 2-3 m/sec. The power available and engine power depend on the flight altitude, and with change of the altitude the vertical climb velocity will also change. The variation of the vertical climb velocity is determined by the altitude characteristics of the engine and is shown graphically (Figure 56a).

For a supercharged reciprocating engine the effective power N_e will increase with increase of altitude from sea level to the critical altitude. The power required also increases. Therefore, the excess power may increase slightly or remain constant up to the critical altitude. After reaching the engine's critical altitude the excess power decreases rapidly. Since the vertical velocity depends on the excess power, it will also decrease.

Using the graph of the variation of engine power and power required for hovering as a function of altitude, we can use (25) to calculate V_y for various altitudes. On the basis of these calculations we can plot the vertical velocity as a function of altitude (Figure 56b), from which we see that the vertical velocity reaches its maximal value at the engine's critical altitude, and then decreases. /84

The altitude at which the vertical climb velocity equals zero is called the helicopter's static ceiling. The static ceiling is the highest altitude at which the helicopter can be hovered. At the static ceiling the excess power $\Delta N = 0$.

However, since both ΔN and ΔV_y approach zero as the helicopter approaches the static ceiling, it is not possible to reach an altitude equal to the theoretical static ceiling. The "practical ceiling" concept has been introduced on this basis. The practical ceiling is the altitude at which the vertical climbing velocity equals 0.5 m/sec.

The static ceiling is defined in terms of rated engine power. A specific power required corresponds to each vertical velocity. Therefore, we can plot the power required for climb as a function of altitude for different vertical velocities (Figure 56c). From this graph we can find the vertical climb velocity at various altitudes for different power required. The altitude characteristic can be used to evaluate the possibility of climbing with a given vertical velocity.

§ 42. Helicopter Vertical Descent With Operating Engine

Helicopter flight downward along a vertical trajectory is termed the vertical descent regime. In this case the following forces act on the helicopter (Figure 57): the helicopter weight G, main rotor thrust force T, and tail rotor thrust $T_{t.r}$.

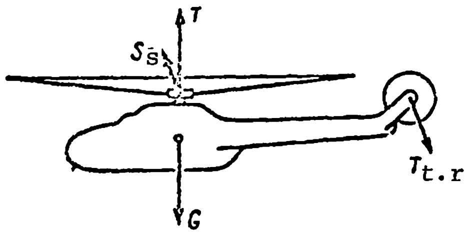

Figure 57. Forces acting on helicopter in vertical descent.

The vertical descent conditions are expressed by the equalities /85

$$T = G;$$

$$T_{t.r} = S_s;$$

$$\Sigma M_{cg} = 0.$$

Here we must bear in mind that we neglect the parasite drag force in view of its small magnitude, and we assume that as a result of the small tilt of the cone axis to the side $T \approx Y$.

In transitioning from the hovering regime to the vertical descent regime, the main rotor pitch must be decreased and in so doing the main rotor thrust force is also reduced. However, as soon as the helicopter transitions into descent the blade element angles of attack increase, which leads to increase of the thrust force to the value present prior to reducing the pitch. Thus, the condition T = G holds for both hovering and vertical descent.

The power required for vertical descent is defined just as in the other vertical regimes

$$N_{des} = N_i + N_{pr},$$

i.e., it is equal to the sum of the induced and profile powers. At constant rpm the profile power is practically independent of main rotor pitch, consequently $N_{pr_{des}} = N_{pr_{hov}}$. The induced power in descent is defined as

$$N_{i_{des}} = T(V_i - V_{des}).$$

During descent less power is required to satisfy the condition T = G than is required in hover.

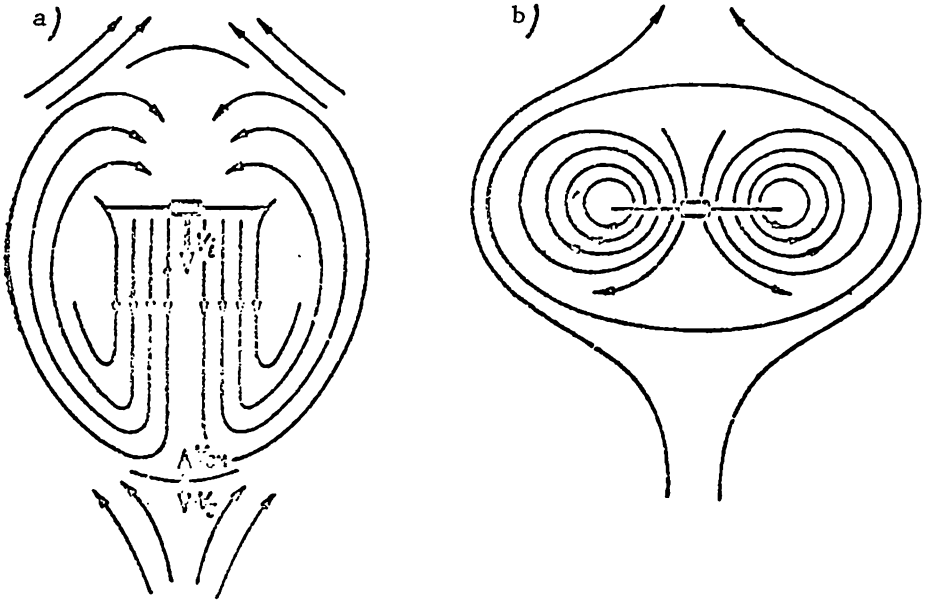

Figure 58. Formation of vortex ring.

§ 43. Vortex Ring Regime

During helicopter vertical descent the air flow accelerated downward by the rotor increases its velocity from V_i to $2V_i$ at a distance from the main rotor equal to about 2R. With further distance from the main rotor the flow decreases its velocity to V_3 as a result of "friction" with the opposing air (Figure 58a).

If the flow velocity V_3 equals the helicopter vertical descent velocity V_{des}, the velocity of the flow accelerated by the rotor $V_3 = 0$, i.e., the /86
rotor essentially "catches" the air which it has accelerated. As a result of inflow above the rotor there will be an "interface" where the rotor essentially "runs away" from the inflowing air with the same velocity. This means that two interfaces are formed: below and above the rotor. At these surfaces the flow leaving the rotor turns and forms closed vortices, which have nearly no effect on thrust formation, since they are far from the rotor.

As the vertical descent velocity is increased, the interfaces where $V_3 = V_{des}$ approach the main rotor. The vortices become more intense and unstable. The rotor expends the power obtained from the engine on rotation of these vortices. The main rotor thrust decreases sharply, since air is not ejected from the closed vortex system (Figure 58b). The vertical descent velocity increases still further. The helicopter begins to toss from side to side, control of the helicopter becomes difficult, and heavy buffeting appears. This flight state corresponds to the developed vortex ring regime. The vortex ring regime occurs with vertical descent at a velocity more than 2-3 m/sec with the engine operating.

The most effective technique for recovery from the vortex ring state is to transition the main rotor into the autorotation regime along an inclined trajectory. But this requires considerable altitude and the absence of obstacles, therefore, the vortex ring regime is dangerous and must be avoided.

Programmed Testing Questions and Answers

Question 1. How is constant height maintained during helicopter hovering? /87

Answer 1. For the hovering height to be constant it is necessary that G = T.

With increase of the specified hovering altitude or with increase of the air temperature the thrust developed by the rotor decreases, therefore, either the main rotor pitch or rpm must be increased, or both may be increased at the same time.

Answer 2. To assure constant helicopter hovering height it is necessary that T = G. With increase of the hovering altitude this condition will be violated; to restore this condition the power supplied to the rotor must be increased.

Answer 3. Constancy of the hovering height requires that T = G, and this is possible when the power required for hovering equals the power supplied to the main rotor from the engine.

Question 2. Effect of change of air parameters on the power required for helicopter hovering.

Answer 1. Increase of the air temperature and pressure leads to increase of its density and, consequently, to increase of the torque required $M_{tor} = m_{tor}F\frac{\rho}{2}(\omega R)^2R$ and increase of the power required.

Answer 2. With increase of the hovering altitude there is a reduction of the air density. Therefore, the power required for hovering increases, since

$$N_r = M_{tor}\omega = m_{tor}F\frac{\rho}{2}(\omega R)^3.$$

Answer 3. Increase of the air temperature and reduction of its pressure lead to reduction of the density, which also decreases with increase of the hover altitude. The power required for hovering equals $N_i + N_{pr}$; the profile power N_{pr} is practically independent of altitude; the induced power equals $N_i = TV_i$. With reduction of the air density there is a reduction of the thrust force $T = 2F\rho V_i^2$, and to maintain the condition $T = G$ it is necessary to increase V_i. Then the power required will also increase.

Question 3. Why does a helicopter have different vertical flight regimes if $T = G$ in all the regimes?

Answer 1. The equality $T = G$ is approximate. It does not take into account the effect of the parasite drag force X_{par}. With account for this force in hovering, climb, and descent $T \approx G + X_{par}$. But the parasite drag force will be different in the different regimes: it will be greatest in a vertical climb and least in a vertical descent. Therefore, the thrust is actually greater than the weight during climb and less during descent; in hovering $T = G + X_{par}$.

Answer 2. All the vertical flight regimes are characterized by absence of vertical acceleration, i.e., constant vertical velocity. According to the law of inertia, the acceleration will be zero if no force acts on the body. The equality $T = G$ is equivalent to the equality $T - G = 0$, which means that it is valid for all regimes.

But in the different regimes the rotor performs different work; in climb the main rotor work $N_i = T(V_i + V_y)$; in hovering $N_i = TV_i$; in descent $N_i = T(V_i - V_{des})$; therefore, more power is required for climbing than for hovering and descent.

Question 4. In what season of the year can a helicopter climb to the highest altitude and lift the greatest load?

Answer 1. The air density is higher in winter than in summer. With increase of the air density the induced power required for hovering decreases while the engine power increases. Consequently, in the winter the excess power $\Delta N = N_{avail} - N_{req}$ is greater, which leads to increase of the static ceiling and lifting capability of the helicopter.

Answer 2. The air density is higher in winter than in summer. Increase of the air density leads to increase of the power required $N_{req} = m_{tor} F \frac{\rho}{2} (\omega R)^3$. This means that the excess power $\Delta N = N_{avail} - N_{req}$ decreases, and this leads to reduction of the static ceiling and lifting capability of the helicopter.

Answer 3. The air density is higher in winter than in summer. With increase of the flight weight it is necessary to increase the main rotor thrust force, but this involves increase of the reactive moment, which will be the larger, the higher the air density. The conclusion is that in the winter the helicopter must develop more power than in the summer, i.e., the helicopter lifting capability and static ceiling decrease in the winter.

CHAPTER VI

HELICOPTER HORIZONTAL FLIGHT

§ 44. General Characteristics of Horizontal Flight /88

Rectilinear flight of a helicopter with constant velocity in the horizontal plane is termed horizontal flight. This is the primary flight regime for the helicopter. Since the Earth is a sphere, flight at constant altitude takes place along a curvilinear trajectory. But the radius of curvature of the Earth's surface is so large that the curvature of the Earth's surface can be neglected in flight.

Only in flights on supersonic airplanes at a speed 2-3 times that of sound is it necessary to consider the Earth's curvature. We shall use an example to demonstrate this. An airplane is flying horizontally at a speed of 1000 m/sec or 3600 km/hr. The airplane weighs 10,000 kgf. Let us find the centrifugal force which arises as a result of curvature of the Earth's surface

$$F_c = \frac{mV^3}{R}$$

where m is the airplane mass, kg;

V is the airplane speed, m/sec;

R is the radius of the Earth, equal to 6 370 000 m.

Then

$$F_c = \frac{10\ 000 \times 1000^2}{6\ 370\ 000} = 1570 \text{ N, or } F_c = 161 \text{ kgf.}$$

We see from the example that the Earth's curvature should be considered when flying at 1000 m/sec, since the airplane lift becomes 161 kgf less than its weight. But for the same flight weight and a velocity of 180 km/hr the centrifugal force is 0.4 kgf.

Therefore, in the following we shall consider the Earth's surface to be flat.

The following forces act on the helicopter in horizontal flight: weight force G, main rotor thrust T, parasite drag X_{par}, and tail rotor thrust $T_{t.r}$ (Figure 59). /89

The horizontal flight conditions are expressed by the equalities

$$Y = G \quad \text{or} \quad G - Y = 0;$$
$$P = X_{par} \quad \text{or} \quad P - X_{par} = 0;$$
$$T_{t.r} = S_s \quad \text{or} \quad T_{t.r} - S_s = 0;$$
$$\sum M_{cg} = 0.$$

The first condition ensures constant flight altitude, the second provides constant velocity, and the third specifies linearity of flight in the horizontal plane.

The forces Y, P, S_s are the components of the main rotor thrust. Accordingly, the main rotor thrust in horizontal helicopter flight performs the functions of propulsive, side, and lifting forces.

§ 45. Thrust and Power Required for Horizontal Flight

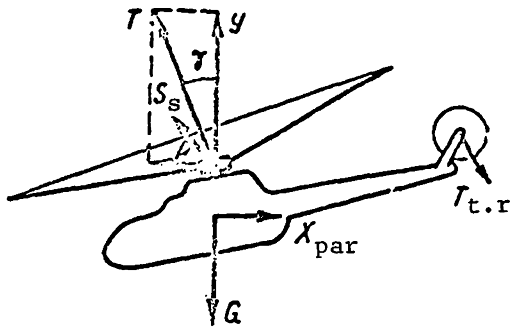

Figure 59. Forces acting on helicopter in horizontal flight.

In horizontal flight the thrust force vector is tilted forward from the vertical and to the side in the direction of the retreating blade.

The result of the lateral tilt of the thrust force vector is the formation of the side force $S_s = T_{t.r}$, and the result of the forward tilt of the thrust force vector is the formation of the propulsive force $P = T \sin \gamma$, which pulls the helicopter forward, overcoming the parasite drag. We recall that helicopter parasite drag is the resistance of all the nonlifting parts (other than the main rotor).

The projection of the thrust force on the vertical yields the lift force $Y = T \cos \gamma$. Therefore, to generate the lift and propulsive forces it is necessary to have the thrust force T_h, which can be found from the force diagram (see Figure 59).

$$T_h = \sqrt{Y^2 + P^2} = \sqrt{G^2 + X_{par}^2}. \tag{26}$$

The thrust required for horizontal helicopter flight depends on its weight and parasite drag, which can be found from the formula

$$X_{par} = C_{x_{par}} F \frac{\rho V^2}{2}.$$

The drag coefficient $C_{x_{par}}$ is found (after wind tunnel tests of the helicopter model) from the formula /90

$$C_{x_{par}} = \frac{2X_{par}}{F\rho V^2}.$$

For the Mi-1 helicopter $C_{x_{par}} = 0.009 - 0.01$.

The parasite drag coefficient depends basically on the shape and attitude of the fuselage, and also on the condition of its surface. The parasite drag is proportional to the flight velocity squared, i.e., $X_{par} = f(V^2)$.

With increase of the horizontal flight velocity the thrust required increases.

In horizontal flight there is a change not only of the magnitude of the thrust required, but also of its direction, i.e., the angle γ of deflection of the thrust force vector from the vertical. Increase of the angle γ is necessary in order to increase the propulsive force P while leaving the lift force Y unchanged.

Tilt of the thrust force vector and increase of the angle are accomplished in three ways:

1) by deflecting the main rotor cone of revolution axis forward;
2) by tilting the helicopter forward;
3) by establishing the main rotor shaft at some angle β relative to the perpendicular to the fuselage structural axis (the line running along the fuselage), Figure 60.

Tilting the cone axis forward through the angle η is accomplished by deflecting the helicopter control stick forward. The main rotor cone axis tilts in the same direction in which the stick is deflected.

Tilting the entire helicopter through the angle θ_h (pitch angle) is accomplished by deflecting the control stick forward. The main rotor shaft

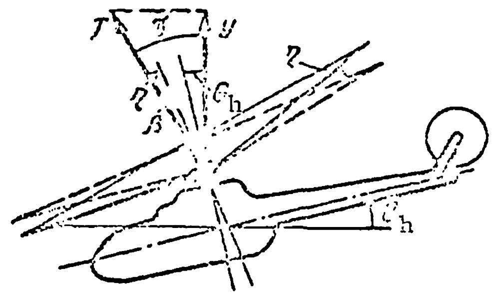

Figure 60. Techniques for tilting thrust force vector.

installation angle relative to the helicopter structural axis always remains the same.

Thus, the thrust force vector forward tilt angle will be equal to the sum of the angles η, θ_h, β. The larger the angle γ, the larger the propulsive force P, and the higher the helicopter speed.

The work which must be supplied to the main rotor shaft per unit time is called the power required for helicopter horizontal flight. The power required is made up of three parts:

1. Motion power, i.e., the work expended per unit time in displacing the helicopter, $N_{mot} = PV$.

2. Induced power, i.e., the work expended per unit time in obtaining the lift force equal to the helicopter weight /91

$$N_i = YV_i = G V_i. \tag{27}$$

3. Power required for overcoming the main rotor blade profile drag

$$N_{pr} = M_{pr}\omega.$$

Using (26a), we find the power required for motion of the Mi-1 helicopter (G = 2200 kgf, H = 0).

$$V = 0;\ N_{mot} = 0;$$

(continued)

$$V = 20 \text{ m/sec}; \quad N_{mot} = \frac{37 \cdot 20}{75} = 11 \text{ hp};$$

$$V = 30 \text{ m/sec}; \quad N_{mot} = \frac{83 \cdot 30}{75} = 37 \text{ hp};$$

$$V = 40 \text{ m/sec}; \quad N_{mot} = \frac{147 \cdot 40}{75} = 79 \text{ hp};$$

$$V = 50 \text{ m/sec}; \quad N_{mot} = \frac{230 \cdot 50}{75} = 153 \text{ hp}.$$

These values make it possible to plot a graph of the variation with flight speed of the power required for motion of the helicopter (Figure 61a). Since N_{mot} and the parasite drag $X_{par} = f(V^2)$, the motion power required $N_{mot} = f(V^3)$ and increases more sharply with increase of the flight speed. The average induced velocity for the main rotor of the Mi-1 helicopter decreases with increase of the horizontal flight speed (Figure 61b).

Using this figure and the Formula (27), we calculate the induced power, i.e., the power expended in creating the helicopter lift force

$$V = 0; \quad N_i = \frac{2200 \cdot 7.5}{75} = 245 \text{ hp};$$

$$V = 10 \text{ m/sec}; \quad N_i = \frac{2200 \cdot 4.5}{75} = 147 \text{ hp};$$

$$V = 20 \text{ m/sec}; \quad N_i = \frac{2200 \cdot 2.6}{75} = 85 \text{ hp}.$$

From these results we can plot the induced power as a function of flight speed (Figure 61c).

The reduction of the induced power with increase of the flight speed is explained by the fact that the rotor interacts with a larger mass of air; therefore, less downwash is required to create a lift force equal to the /92

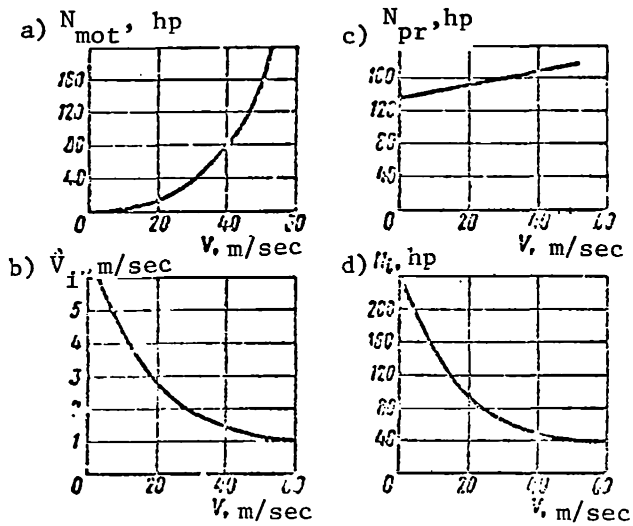

Figure 61. Power components versus flight speed.

helicopter weight. The power expended in overcoming profile drag increases with increase of the flight speed (Figure 61d). The increase of the profile power is explained by the increase of the air friction forces in the blade boundary layer with increase of the flow velocity over the blades. In the forward flight regime the flow relative velocity increases for the advancing blades ($W = u + V \sin\psi$), while it decreases for the retreating blades ($W = u - V \sin\psi$), but since $X_{pr} = f\ (W^2)$ the drag of the advancing blade increases more rapidly than the drag of the retreating blade decreases. After calculating the component parts of the power required, we find the power required for horizontal flight of the Mi-1 helicopter

$$N_h = N_{mot} + N_i + N_{pr};$$

$$V = 0;\ N_h = N_{mot} = 381 \text{ hp};$$

(continued)

$$V = 10 \text{ m/sec}; \ N_h = 4 + 147 + 140 = 291 \text{ hp};$$

$$V = 30 \text{ m/sec}; \ N_h = 37 + 62 + 160 = 259 \text{ hp};$$

$$V = 50 \text{ m/sec}; \ N_h = 153 + 42 + 180 = 375 \text{ hp}. \qquad (28)$$

Using these values, we plot the power required for horizontal flight as a function of the flight speed for the Mi-1 helicopter ($H = 0$; $G = 2200$ kgf) (Figure 62). We see from this figure that with increase of the velocity from zero to 80 km/hr the power required for horizontal flight decreases. The speed for which the power required for horizontal flight is minimal is called the helicopter's economical speed.

§ 46. Characteristic Helicopter Horizontal Flight Speeds /93

The characteristic horizontal flight speeds define to a considerable degree the helicopter flight qualities. Calculation of these speeds, and then their verification in flight, is one of the important problems of helicopter aerodynamic design. The calculation of the characteristic speeds is most often made using the power method, first suggested by Zhukovskiy.

To construct the power required and available curves we use the curve of Figure 62. We find the power available at rated and takeoff engine rpm from the formulas

$$N_{avail} = N_e \zeta$$

$$N_{avail_{rat}} = 430 \cdot 0.78 = 336 \text{ hp};$$

$$N_{avail_{to}} = 575 \cdot 0.78 = 448 \text{ hp};$$

We use these values to plot a curve (Figure 63a), which permits determining the following characteristic horizontal helicopter flight speeds:

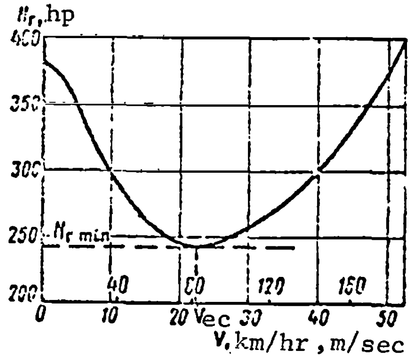

Figure 62. Power required versus flight speed.

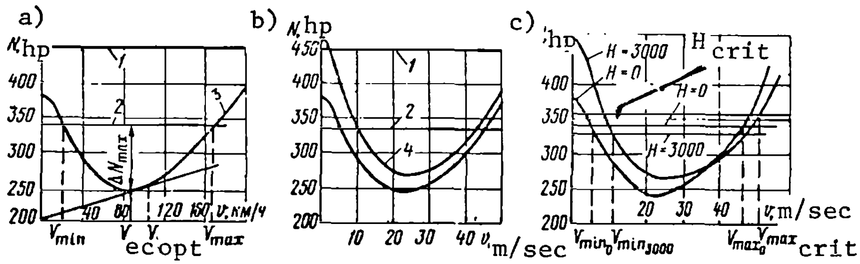

Figure 63. Power required and available as a function of helicopter weight and flight altitude:

1) N_a for H = 0, n = 2200 rpm;
2) N_a for H = 0, n = 2050 rpm;
3) N_h for H = 0, G = 2200 kgf;
4) N_h for H = 0, G = 2300 kgf.

1. The maximal speed, which corresponds to the point of intersection of the power required and available curves. When using rated engine power, the maximal horizontal flight speed at sea level will be about 165 km/hr; when using takeoff power, this speed will be 208 km/hr.

2. The optimal velocity at which the longest flight range is obtained. This speed corresponds to the point of contact of a tangent drawn from the coordinate origin to the power required curve (for the Mi-1, V_{opt} = 90-95 km/hr).

3. The economical speed, i.e., the speed corresponding to minimal power required (for the Mi-1, V_{ec} = 80 km/hr).

4. The minimal speed. When using rated power at a flight weight of 2200 kgf, the helicopter cannot hover at sea level. It can only perform horizontal flight with a speed of about 20 km/hr. However, heavy vibration develops at low speed; therefore, the constructor has established speed limitations: minimal permissible is 40 km/hr, maximal permissible is 170 km/hr. /94

5. The horizontal flight speed range, i.e., the speeds at which horizontal flight is possible. When using rated power the speed range is

$$\Delta V_{rat} = V_{max} - V_{min} = 165 - 20 = 145 \text{ km/hr}.$$

When using more than rated power, the total speed range is $V_{tot} = 170 - 0 = 170$ km/hr.

6. The excess power, i.e., the difference between the power available and required for horizontal flight at a given speed. Each speed is associated with a given excess power $\Delta N = N_{avail} - N_h$. The maximal excess power will occur for flight at the economical speed.

§ 47. Effect of Helicopter Weight and Flight Altitude on Performance

With increase of the helicopter weight there is an increase of the power required for horizontal flight, since $N_l = \frac{GV_l}{75}$. Figure 63b shows curves of the power required for the Mi-1 helicopter for flight weights of 2200 kgf and 2300 kgf. In comparing these curves we can say that with increase of the flight weight:

the maximal horizontal flight speed decreases;

the minimal speed when using rated power increases;

the economical and optimal speeds increase, although only slightly;

the horizontal flight speed range decreases;

the excess power decreases;

hovering of the helicopter outside the air cushion influence zone is impossible even when using takeoff power.

These variations of the helicopter flight characteristics should always be taken into account, particularly in those cases when a large fuel supply is carried. If the flight performance is based on takeoff weight, the values obtained will be too low. Therefore, if a large fuel supply is carried, the flight performance is based on the average flight weight with consideration for the fuel consumption

$$G_{av} = G_{to} - \frac{G_{fuel}}{2}$$

where G_{av} is the average flight weight; /95

G_{to} is the takeoff flight weight;

G_{fuel} is the fuel weight (tanks completely full).

Effect of flight altitude. The helicopter flight characteristics depend on the flight altitude and also on the air temperature and humidity. The air density decreases with increase of the altitude; therefore, the parasite drag decreases, as does the power required for motion

$$N_{mot_H} = N_{mot_0} \Delta$$

where

$$\Delta = \frac{\rho_H}{\rho_0}.$$

Since the power required for motion has a large value at speeds above the economical speed, a change of flight altitude will have an effect on this speed

$$N_t = YV_t = GV_t.$$

In studying the hovering regime, it was established that the thrust developed by the main rotor depends on the flight altitude, i.e., this thrust decreases with increase of the altitude, and this means that the lift force will decrease. But since the horizontal flight conditions specify that Y = G, it is necessary to increase the induced velocity V_i. Consequently, the induced power $N_i = GV_i$ will increase in proportion to $1/\Delta$, i.e., $N_{iH} = N_{i_0} \frac{1}{\Delta}$. The profile power changes very little with increase of the altitude.

Thus, with increase of the altitude the power required for motion decreases, while that required for creating the lift force increases. These conclusions are illustrated by the plot of power required for different altitudes (Figure 63c). This figure shows also how the power available varies with altitude.

For the supercharged engine the effective power increases up to the critical altitude and then decreases. As a result of this variation of the power available and the variation discussed above of the motion power and the induced power, we can say that with increase of the altitude up to the critical altitude:

1. For speeds lower than optimal, the power required for horizontal flight increases owing to the increase of the induced component of this power.

2. For speeds above optimal, the power required for horizontal flight decreases as a result of decrease of the motion power.

3. The magnitude of the optimal speed changes very little with change of the flight altitude.

4. The maximal and minimal horizontal flight speeds increase.

5. The excess power increases up to the engine critical altitude and then decreases.

Consequently, if flight must be accomplished at high speed, this should be done at high altitude. /96

Increase of the air temperature is equivalent to increase of the altitude, since the air density decreases as its temperature increases. Increase of the air humidity leads to reduction of engine power and of the maximal horizontal flight speed.

All these conclusions are valid if we ignore the factors which restrict the maximal horizontal flight speed.

§ 48. Factors Limiting Maximal Horizontal Flight Speed and Ways to Increase This Speed

It was established above that to increase the flight speed we must increase the angle γ of deflection of the thrust force vector (see Figure 60) and the magnitude of the main rotor thrust force. The thrust force can be increased in two ways: by increasing the angular velocity ω; and by increasing the main rotor pitch, since this leads to increase of the thrust coefficient c_T (see Figure 16).

Increase of the main rotor thrust force by increasing the rpm involves increase of the blade drag profile, and consequently increase of the fuel consumption per unit rotor thrust developed. This approach is not advisable. Moreover, increase of the thrust by increasing the rotor rpm is possible only up to a definite limit.

It is well-known that increase of the angular velocity increases the circumferential velocity $u = \omega R$. In the forward flight regime the resultant

blade element velocity is $W = u + V \sin \psi$, i.e., at the azimuth 90° $W = u + V$. With increase of the rpm there is an increase of the circumferential velocity and the flight velocity; consequently, the resulting velocity W will increase. Increase of the resultant velocity of the flow past the blade elements is permissible only until the velocity reaches the critical value, i.e., until the appearance on the blade of a local flow velocity equal to the speed of sound (compression shocks develop, and blade shock stall manifests itself).

Thus, the first technique for increasing main rotor thrust by increasing the rpm is limited by the appearance of shock stall at the tip of the blade when the blade is located at the 90° azimuth.

Let us examine the second approach. Increase of main rotor thrust by increasing the pitch and the coefficient c_T involves increase of the blade element angles of attack. It is well-known that in the forward flight regime the blade element angles of attack vary in azimuth: the angles of attack are smallest at the 90° azimuth, and they are largest at the 270° azimuth. The higher the flight speed, the larger the angles of attack of the blade tip elements at the 270° azimuth. Increase of the pitch leads to further increase /97 of the angles of attack. If the angles of attack at the tip elements approach the critical value, flow separation analogous to the separation from an airplane wing is formed at the end of the blade (Figure 64).

Moreover, with increase of the flight velocity the reverse flow zone expands; in accordance with the formula $d = \mu R$, the diameter of this zone increases with increase of the flight speed at constant rpm. Expansion of the stall separation and reverse flow zones leads to reduction of the main rotor thrust force and causes severe roughness. Flight cannot be continued under separated flow conditions, therefore, the flight speed can be increased only until the angles of attack of the blade tip elements become close to the critical value. This flight speed limitation is called the 270°-azimuth blade stall limitation. Thus, the maximal speed for the Mi-1 helicopter is limited to 170 km/hr, while the Mi-4 is limited to 175 km/hr.

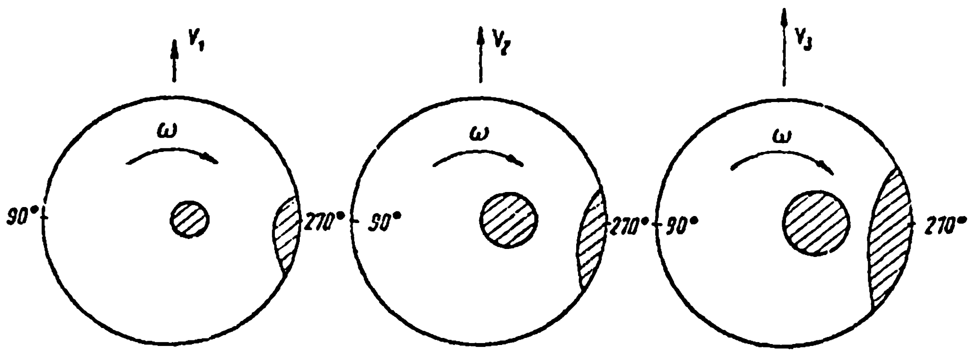

Figure 64. Flow separation from blades with increase of flight speed.

If these limitations did not exist, the power available would permit a maximal speed at altitude of 205 km/hr for the Mi-1 and a speed of 225 km/hr at an altitude of 1500 m for the Mi-4.

How can the maximal flight speed of the helicopter be increased?

In order to avoid blade tip stall at the 270° azimuth with increase of the forward speed, it is necessary to reduce the main rotor pitch. But this leads to reduction of the thrust and creation of a deficiency of the lift force ($-\Delta Y$) and of the propulsive force ($-\Delta P$) (Figure 65). In order to avoid these phenomena, use is made of an auxiliary wing, which creates the additional lift force ΔY.

The additional propulsive force ΔP necessary to increase the speed is created either by increasing the engine thrust or by the thrust of additional propellers. Helicopters with an additional wing and additional propellers are called compound helicopters. In these helicopters there is an unloading of the main rotors such that they can operate with lower incidence angles, and blade stall occurs at higher speeds. Helicopters of this type include the /98

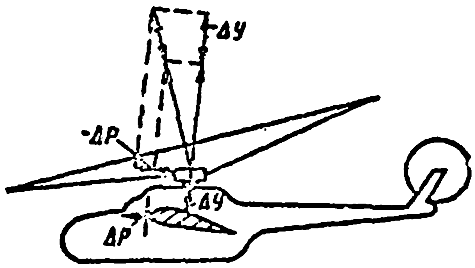

Figure 65. Techniques for unloading main rotor.

Mi-6 and Kamov's prop-wing design with side-by-side rotors. The speed of the compound helicopters reaches 400-450 km/hr. However, they also have drawbacks: increased structural weight and increased parasite drag as a result of the slipstream from the main rotor flowing over the wing. This situation is particularly characteristic for the vertical flight regimes with low horizontal velocities.

§ 49. Horizontal Flight Endurance and Range

Horizontal flight endurance is the time in the course of which the helicopter can perform horizontal flight using the available fuel supply. Flight endurance is found from the formula

$$T_{hr} = \frac{G_{fuel}}{c_h}$$

where G_{fuel} is the fuel supply for horizontal flight, liters;
c_h is the fuel consumption per hour, liters/hr.

This formula shows that the endurance depends on the fuel supply and the hourly consumption. The fuel supply for horizontal flight G_{fuel} is the difference between the amount of fuel serviced into the tanks G_t and the amount of fuel expended in the other flight regimes: taxiing, takeoff, climb, descent, and landing. The fuel consumption in these flight regimes is indicated in the instructions for calculating flight endurance and range, which are prepared for each helicopter type on the basis of calculations and flight tests.

The hourly fuel consumption is the amount of fuel which the engine consumes per hour of operation. It is found from the formula

$$c_h = c_e N_e = c_e \frac{N_r}{\zeta} = \frac{c_e}{\zeta} N_h, \tag{29}$$

where c_e is the specific fuel consumption;

N_e is the effective engine power;

ζ is the power utilization coefficient.

Since c_e and ζ change only slightly with variation of the flight speed, their ratio can be assumed constant and (29) takes the form

$$c_h = \text{const}\, N_h.$$

This formula shows that the hourly fuel consumption depends on the power required for helicopter horizontal flight, and consequently, on the flight speed. /99

Using the curve of power versus speed (see Figure 62), we can say that the minimal power required for horizontal flight corresponds to the economical speed; therefore, the minimal hourly fuel consumption corresponds to this speed.

In order for the helicopter to stay in the air for the maximal time, flight must be performed at the economical speed. The economical speed depends on helicopter weight: this speed increases with increase of the weight, and the flight endurance decreases. Since the economical speed changes very little with altitude, the horizontal flight endurance decreases somewhat with increase of the altitude as a result of the increased fuel consumption in climb and descent.

Helicopter horizontal flight range is the distance which the helicopter can fly while utilizing the fuel supply for horizontal flight

$$L_h = \frac{G_{fuel}}{c_k} \tag{30}$$

where c_k is the fuel consumption per kilometer, liters/km.

The horizontal flight range is larger, the larger the fuel supply and the lower the consumption per kilometer. The fuel supply is defined as in the flight endurance calculation. The consumption per kilometer is found from the formula

$$c_к = \frac{c_h}{V} \doteq = \text{const}\,\frac{N_г}{V}.$$

The minimal fuel consumption per kilometer will be achieved with the minimal ratio N_h/V.

Let us examine the power required and available curves for horizontal flight (Figure 66). Any point on the power required curve corresponds to definite values of V and N. For example, the point 1 corresponds to the speed V_1, and the power required N_1. The ratio of these quantities is equal to tg γ, and therefore, the consumption per kilometer is

$$c_к = \text{const}\,\text{tg}\,\gamma_1.$$

This means that tg γ must be minimal in order to obtain the minimal fuel consumption per kilometer. This corresponds to the angle between the tangent to the power required curve and the horizontal axis. The point of tangency will correspond to the optimal helicopter horizontal flight speed.

Thus, the maximal horizontal flight range is achieved at the optimal speed, which corresponds to the minimal fuel consumption per kilometer only if the engine is carefully adjusted. In this case, the calculation of the maximal range is made using the fuel consumption per kilometer curves plotted /100

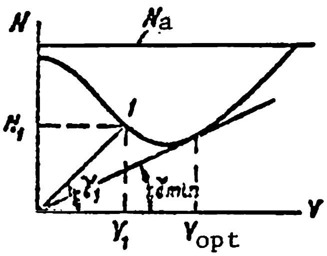

Figure 66. Dependence of N/V on flight speed.

on the basis of experimental helicopter operational data. The speed obtained from these curves and the corresponding minimal consumption will be close to the optimal values.

The consumption per kilometer is the fuel burned per kilometer of air distance (relative to the air). Consequently, the flight range calculation made using (30) is valid only if there is no wind. If there is wind, the flight range will vary, depending on the wind direction and velocity. The so-called navigational fuel supply, amounting to 10-15% of the required fuel, is set aside as reserve in case the weather conditions change. Since the power required for horizontal flight depends on helicopter weight and flight altitude, the fuel consumption per kilometer increases, and the range decreases with increase of the weight. The average flight weight is used for exact range calculations

$$G_{av} = G_{to} - G_{fuel}1/2$$

where G_{to} is the takeoff weight;

G_{fuel} is the fuel supply for horizontal flight.

With increase of the altitude the optimal speed increases somewhat, and the power required decreases, therefore, the fuel consumption per kilometer also decreases. But the climb to a higher altitude requires more fuel. In practice, the longest flight range without account for wind is obtained at an altitude from 1000 to 2000 meters. For the Mi-1, the minimal fuel consumption per kilometer is 0.56 liters/km at an altitude of 1000 meters and an indicated airspeed of 130 km/hr. The flight range with this fuel consumption rate is 370 km.

Programmed Testing Questions and Answers

Question 1. How does the main rotor thrust developed vary with increase of the helicopter horizontal flight speed?

Answer 1. The thrust required increases with increase of the horizontal flight speed. Therefore, the thrust available must be increased to increase the speed. Moreover, the tilt of the thrust force vector relative to the vertical must be increased in order to increase the propulsive force P while leaving the lift force Y unchanged.

Answer 2. The thrust required for horizontal flight increases with increase of the speed. The thrust available also increases with increase of the flight speed. Therefore, the magnitude of the thrust available must be left unchanged while increasing the tilt angle γ relative to the vertical.

Answer 3. During hover and in horizontal flight at any speed, the thrust developed by the rotor must be equal to the thrust required. The thrust available, or the thrust which the main rotor develops at constant power supplied to the rotor, increases more rapidly with increase of the speed than does the thrust required. Therefore, with increase of the flight speed up to the economical speed the thrust available must be decreased, while upon further increase of the flight speed the thrust available must be increased. At the same time, with increase of the flight speed the tilt of the thrust force vector relative to the vertical must be increased. /101

Question 2. How must the engine power be varied with change of the flight speed?

Answer 1. With increase of the flight speed there is an increase of the power required for motion and the power required for overcoming profile drag. The induced power decreases. Therefore, the engine power remains constant.

Answer 2. The power required and the power developed must be equal for any flight speed. With increase of the speed from 0 to V_{ec}, the power required decreases; therefore, in this range the engine power developed must be reduced. With increase of the speed from V_{ec} to V_{max} the power required increases, therefore, the power developed must also be increased.

Answer 3. With increase of the flight speed the helicopter parasite drag increases as the speed squared, while the power required for motion increases as the speed cubed. The profile drag power also increases with increase of the speed, while the induced power decreases only slightly. Consequently, the power developed by the engine must be increased with increase of the flight speed.

Question 3. How does helicopter performance vary with change of the flight weight?

Answer 1. With increase of the flight weight, the power required to develop the lift force Y = G increases. However, to increase the lift force, the thrust force vector tilt must be reduced. As a result of this, there is a reduction of the helicopter parasite drag, which in turn leads to increase of the maximal horizontal flight speed and increase of the speed range.

Answer 2. Increase of the flight weight leads to increase of the thrust force required. But since the thrust developed by the main rotor increases faster than the required thrust with increase of the speed, the engine power developed must be reduced. Therefore, the maximal horizontal flight speed, speed range, and excess power increase with helicopter weight increase.

Answer 3. With increase of the flight weight the power required for helicopter horizontal flight increases as a result of increase of the induced power $N_i = GV_i$. The power available is independent of helicopter weight. The power required curve is shifted upward. As a result, the maximal horizontal flight speed is reduced, and the minimal horizontal flight speed is increased. The speed range and the excess power decrease.

Question 4. Effect of flight altitude and air temperature on helicopter flight characteristics.

Answer 1. Increase of the flight altitude involves reduction of the air density. This leads to reduction of the thrust developed by the main rotor. Since the horizontal flight condition is the equality G = Y, with increase of the altitude the induced velocity V_i must be increased; therefore, the induced power $N_i = GV_i$ will increase. The power required curve is shifted upward for low flight speeds.

At high flight speeds, the induced power increases only slightly, while the power required for motion decreases with increase of the altitude ; therefore, at high speeds the power required curve is shifted downard. As a result of this change of the power required and available, the minimal and maximal horizontal flight speeds increase up to the engine critical altitude. Above the critical altitude the maximal flight speed will decrease. Increase of /102
the air temperature is equivalent to increase of the altitude.

Answer 2. With increase of the altitude the power required for helicopter horizontal flight increases. The power developed by the engine decreases. The power required curves are shifted upward and the power available curves are shifted downward. As a result of this displacement of the curves, the maximal speed decreases and the minimal increases; the speed range and the excess power decrease. Increase of the air temperature leads to reduction of the power required and available.

Question 5. Factors limiting maximal helicopter horizontal flight speed and ways to increase this speed.

Answer 1. The maximal horizontal flight speed of a helicopter is limited by the engine power available. Flight at speeds higher than the maximal is not possible, since more power is required for such flight than the engine develops. Powerful gas turbine engines are installed on the new helicopters, for example, the Mi-2, Mi-6, and Mi-8, to increase the maximal flight speed.

Answer 2. With increase of the horizontal flight speed, the forward tilt of the main rotor plane of rotation must be increased. This leads to reduction of the main rotor angle of attack. As a result of the reduction of the angle of attack, the rotor thrust force decreases. This is then the reason for the maximal speed limitation. To increase the maximal speed we must increase the main rotor rpm, which requires the more powerful gas turbine engines.

Answer 3. The rotor thrust force must be increased in order to increase the flight speed. If the thrust force is increased by increasing the rpm, local compression shocks develop on the blades. If we increase the thrust force by increasing the main rotor pitch, then the angles of attack of the blade elements at the 270° azimuth increase. The speed is limited by the onset of blade stall at angles of attack above the critical value. To increase the speed the main rotor must be unloaded by installing an additional wing or thrusting propellers, i.e., compound helicopters such as the Mi-6, Kamov rotor-wing, and so on must be constructed.

CHAPTER VII

CLIMB ALONG INCLINED TRAJECTORY

§ 50. General Characteristics of the Climb Regime Along an Inclined Trajectory /10

Climb along an inclined trajectory is rectilinear flight of the helicopter with constant velocity and constant angle relative to the horizontal plane.

In this flight regime the helicopter is subject to the weight force, main rotor and tail rotor thrust forces, and the parasite drag force (Figure 67). To determine the flight conditions the helicopter weight force /10 can be broken down into components: G_1 perpendicular to the flight path, and G_2 parallel to the flight path and directed opposite the motion.

The main rotor thrust force can also be resolved into the components Y (lift force), P (propulsive force), S_s (side force). The conditions for steady climb are expressed by the equalities

$Y = G$ — (rectilinear flight and constant climb angle);

$P = G_2 + X_{par}$ — (constant velocity);

$T_{t.r} = S_s$ — (absence of lateral displacment of the helicopter);

$\Sigma M_{cg} = 0$ — (absence of helicopter rotation about its center of gravity).

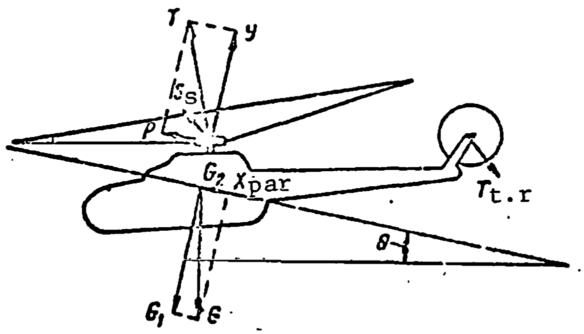

Figure 67. Forces acting on helicopter in climb.

§ 51. Thrust and Power Required for Climb

The thrust required for climb can be found by using the diagram of the forces acting on the helicopter (Figure 67),

$$T_{cl} = \sqrt{Y^2 + P^2} = \sqrt{G_1^2 + (X_{par} + G_2)^2} \tag{31}$$

where

$$G_1 = G\cos\theta; \quad G_2 = G\sin\theta.$$

Comparing (31) with (26), we can say that the helicopter parasite drag in climb is practically equal to the parasite drag in horizontal flight at the same speed. For example, for the Mi-1 in climb (V_{cl} = 85 km/hr; $\theta \approx 9°$) the parasite drag X_{par} = 55 kgf, and in horizontal flight X_{par} = 47 kgf. The slight increase is explained by the fact that during climb the air flow approaches the fuselage at a large negative angle, which leads to increase of the parasite drag force coefficient from 0.009 to 0.0097. The lift force is less during climb than in horizontal flight. The propulsive force during climb will be greater than the propulsive force in horizontal flight by the

magnitude G sin θ. Consequently, in (31) the first term of the radicand is smaller than in (26), while the second term is larger than the second term of (26). Therefore, the thrust required for climb along an inclined trajectory is practically the same as the thrust required for horizontal flight at the same speed.

If a helicopter can hover, then it can climb along an inclined trajectory. This conclusion is confirmed by the fact that excess thrust appears in the forward flight regime (Figure 68). Consequently, even with some thrust deficiency for hovering (dashed curve in Figure 68), climb along an inclined trajectory at a speed greater than the minimal horizontal flight speed is /104
possible. This circumstance is utilized in the running helicopter takeoff.

The power required for climb is found from the same formula as used to find the power required for horizontal flight

$$N_{cl} = N_{mot} + N_i + N_{pr}.$$

But in this formula the terms may differ from the corresponding terms for the power required for horizontal flight.

During climb N_{pr} will not differ from the profile power for horizontal flight if the rpm and flight speed are the same.

The induced power in climb is practically equal to the induced power for horizontal flight, since

$$N_i = YV_i, \text{ and } Y_{cl} = G \cos \theta \approx G.$$

But the power required for motion during climb differs considerably from the motion power for horizontal flight

$$N_{mot} = PV_{cl} = (X_{par} + G_2)\, V_{cl} = X_{par}V_{cl} + G_2V_{cl}.$$

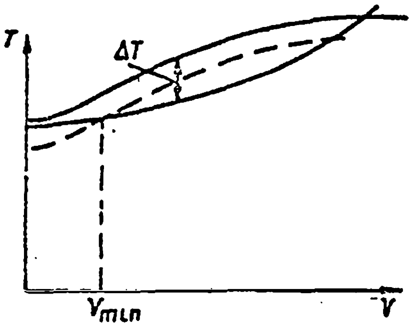

Figure 68. Thrust required and available versus flight speed.

If $V_{cl} = V_h$, then $X_{par}V_{cl} \approx N_{mot_h}$, since

$$X_{par_{cl}} = X_{par_h}$$

then, equating

$$G_2 V_{cl} = \Delta N$$

we obtain

$$N_{mot_{cl}} = N_{mot_h} + \Delta N.$$

Climb is possible if there is excess power ΔN (i.e., the power available exceeds the power required for horizontal flight)

$$N_{cl} = N_h + \Delta N. \tag{32}$$

§ 52. Vertical Rate of Climb

During climb the helicopter center of gravity moves along the flight trajectory with the velocity V_{cl}. The climbing velocity vector can be represented as the vector sum

$$\overline{V}_{cl} = \overline{V}_h + \overline{V}_y,$$

where $\overline{V}_h$ is the horizontal component of the velocity;
$\overline{V}_y$ is the vertical rate of climb.

The horizontal component of the velocity equals $V_{cl} \cos \theta$, but since the climb angle is usually small and does not exceed 10°-12°, then $V_h \approx V_{cl}$. /105

This means that the same power is expended on horizontal displacement of the helicopter during climb as is expended in horizontal flight at the same speed.

Analyzing (32) from this viewpoint, we can say that the excess power $\Delta N = GV_y$ is expended on vertical displacement of the helicopter. Hence, knowing the helicopter weight and the excess power, we find the vertical rate of climb

$$V_y = \frac{\Delta N}{G}.$$

The excess power can be found from the power required and available curves for helicopter horizontal flight (see Figure 63a). The maximal excess power corresponds to the economical speed (for the Mi-1, V_{ec} = 80 km/hr or about 23 m/sec). Therefore, climbing should be performed at the economical speed. Moreover, from the power required and available curves for the Mi-1, we can conclude that vertical climb is impossible when using rated engine power. This means that the static ceiling when using rated power is equal to zero, i.e., helicopter hovering is possible only in the air cushion influence zone.

When using takeoff power vertical climb is possible, but the rate of climb will be lower than when climbing along an inclined trajectory. Consequently, this once again confirms that climbing should be performed along an inclined trajectory. Vertical climbing is performed only when it is necessary to clear surrounding obstacles. It must be kept in mind that takeoff power can be used only for a brief period, not to exceed five minutes.

Increase of the flight altitude involves change of the power required and the power available, and,therefore, change of the vertical rate of climb.

§ 53. Variation of Vertical Rate of Climb with Altitude

If we calculate the power required for horizontal helicopter flight at various altitudes and construct curves of these powers, and if we find the power available at various altitudes from the engine altitude characteristics,

then we can use these curves to draw important conclusions on altitude variation of the helicopter flight characteristics.

Let us examine such curves for the Mi-1 helicopter (Figure 69a). We see from the curves that: /106

1) for flight speeds less than optimal the power required curves are shifted upward;
2) for flight speeds greater than optimal these curves are shifted downward;
3) the power available lines shift upward up to the critical altitude of 2000 meters and then shift downward; this shift causes increase of the maximal speed up to the critical altitude and reduction at altitudes above critical;
4) there is an increase of the minimal speed and an initial increase and subsequent decrease of the excess power (see Table). We obtained these data using rated engine power. If takeoff power is used, vertical climb can be performed up to an altitude of about 1000 m, and the maximal speed at sea level will be about 210 km/hr.

ALTITUDE VARIATION OF FLIGHT CHARACTERISTICS

H, M	V_{max}, km/hr	V_{min}, km/hr	ΔN, hp	V_y, km/hr
0	166	18	100	3,4
2000	180	25	110	3,8
3000	169	40	63	2,2
4000	150	58	30	1

The tabular data can be used to plot two graphs which characterize the helicopter flight characteristics:

altitude dependence of the vertical rate of climb;

altitude dependence of the maximal and minimal speeds.

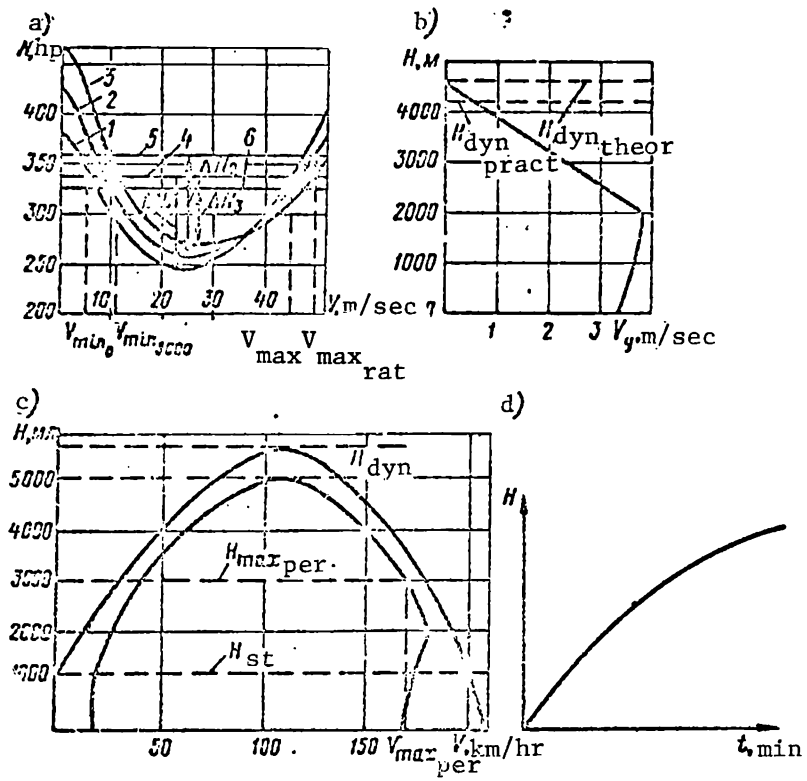

Figure 69. Helicopter aerodynamic characteristics:

1, 2, 3 — N_h; 4 — N_{avail} for H = 0; 5 — N_{avail} for H = 2000 m; 6 — N_{avail} for H = 3000 m.

The plot of vertical rate of climb versus altitude (Figure 69b) shows that the vertical rate of climb increases up to the engine critical altitude. Above this altitude the rate of climb decreases.

The altitude at which the vertical rate of climb for flight along an inclined trajectory equals zero is called the helicopter dynamic ceiling. More precisely, this altitude is called the theoretical dynamic ceiling. Since the helicopter does not actually climb to this altitude, the practical

ceiling concept is introduced, at which the vertical rate of climb equals 0.5 m/sec. The maximal and minimal speed plot (Figure 69c) shows the increase of the helicopter maximal horizontal flight speed with altitude increase from zero to the engine critical altitude. At altitudes above critical the maximal speed decreases. The minimal speed increases with increase of the altitude. At the dynamic ceiling altitude, the helicopter can perform flight only at a /107
single speed, which will be both maximal and minimal at the same time. During flight at altitudes less than the dynamic ceiling, the helicopter has a range of speeds in horizontal flight from minimal to maximal.

Figure 69c shows two curves: one of them corresponds to flight using rated engine power; the other corresponds to use of takeoff engine power. In the latter case, we see the helicopter static ceiling, i.e., the maximal altitude for helicopter hovering out of the air cushion effect.

The graph showing the variation of the maximal and minimal horizontal flight speeds as a function of altitude is called the helicopter aerodynamic "passport". This "passport" characterizes the helicopter flight data. In many cases, the flight characteristics have various limitations, which ensure structural strength or an acceptable vibration level. Thus, for the Mi-1 helicopter, the maximal flight speed must not exceed 170 km/hr at altitudes from 0 to 3000 meters.

The ceiling of the Mi-1 is limited to an altitude of 3000 meters. /108

Record speeds for light helicopters ($V = 210$ km/hr and $H \approx 6000$ m) have been established during flight tests and in special flights on the Mi-1 helicopter.

If the vertical rate of climb is used to calculate the time to climb to various altitudes, we can plot the so-called climb barogram, which also characterizes the helicopter flight characteristics (Figure 69d).

CHAPTER VIII

HELICOPTER DESCENT ALONG INCLINED TRAJECTORY

§ 54. General Characteristics of the Descent Regime /108

Rectilinear flight at constant velocity along an inclined trajectory is termed the helicopter descent regime with operating engine. A characteristic of this regime is the possibility of controlling the vertical rate of descent and the speed along the trajectory by varying the power supplied to the main rotor.

In this regime the following forces act on the helicopter: weight, main rotor thrust, parasite drag, and tail rotor thrust (Figure 70).

The helicopter motion takes place along a trajectory which is inclined to the horizon at the angle θ, termed the descent angle.

We resolve the weight force G and the main rotor thrust force T into components perpendicular and parallel to the flight trajectory. We obtain the weight force components $G_1 = G \cos \theta$ and $G_2 = G \sin \theta$. The main rotor thrust components will be the lift force Y perpendicular to the flight trajectory, and the force P_x parallel to this trajectory. The force P_x may be directed either opposite the helicopter motion direction or in the direction of this motion.

The direction of the force P_x depends on the position of the cone axis and the main rotor plane of rotation. If the cone axis is perpendicular to

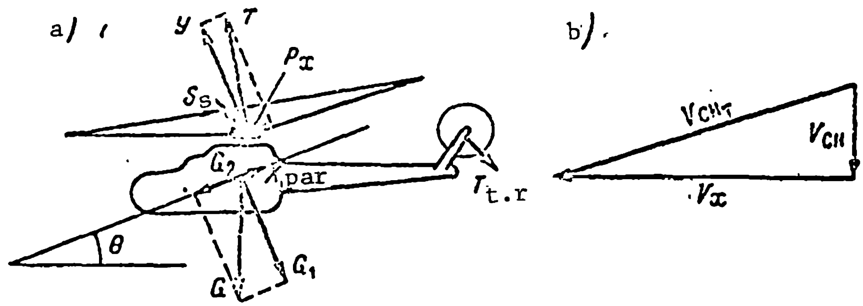

Figure 70. Forces acting on helicopter in descent.

the trajectory, then $P_x = 0$. If the cone axis is inclined aft relative to the perpendicular, then P_x will be directed opposite the helicopter motion and will retard this motion. If the cone axis is tilted forward, the force P_x will be directed along the motion and together with the component G_2 will be a propulsive force. The cone axis direction is connected with the position of the rotor plane of rotation and, consequently, with the main rotor angle of attack. Most frequently, the main rotor angle of attack is close to zero or has a small negative value. During flight with a large descent angle, the angle of attack is positive and the force P_x is directed opposite the motion.

Steady state descending flight is possible under the following conditions

$$Y = G_1 = G \cos \theta; \quad (33)$$

$$G_2 = X_{par} \pm P_x; \quad (34)$$

$$T_{t.r} = S_s;$$

$$\sum M_{cg} = 0.$$

The first condition assures rectilinear flight and constant descent angle. Consequently, by varying the lift force Y we can alter the helicopter descent angle. When the lift force is increased, the descent angle decreases, and vice versa. The second condition assures constant helicopter speed. Let us compare these conditions with those for climb along an inclined trajectory.

The first condition is the same for descent and climb. The second conditions differ fundamentally from one another: in climb, the propulsive force is the main rotor thrust component P, while in descent this force will be the weight force component G_2. The thrust force component P_x may be either a part of the propulsive force or a part of the retarding force, depending on the position of the main rotor cone axis. The third and fourth descent conditions are analogous to the same conditions for the other flight regimes.

§ 55. Thrust and Power Required for Helicopter Descent

The thrust developed by the main rotor during flight along an inclined trajectory must provide the necessary magnitudes of the lift force Y and its component P_x parallel to the motion trajectory. In accordance with the diagram of the forces acting on the helicopter, the thrust force $T = \sqrt{Y^2 + P_x^2}$. /110
From (33) we find $Y = G_1 = G\cos\theta$, and from (34) $\pm P_x = G_2 - X_{par} = G\sin\theta - X_{par}$, then

$$T = \sqrt{(G\cos\theta)^2 + (G\sin\theta - X_{par})^2}. \tag{35}$$

Since in most cases the descent angle θ is small (less than 10°) $\cos\theta \approx 1$. This means that the first term of the radicand in (35) is close to one and the second term is close to zero. Hence, we can conclude that the thrust required for helicopter descent along an inclined trajectory will be practically equal to the helicopter weight. For large descent angles, when the angle θ approaches 90°, the first term of the radicand of (35) approaches zero, and the difference $G\sin\theta - X_{par}$ approaches in magnitude the weight, i.e., $T \approx G$.

We came to the same conclusion in studying helicopter vertical descent. Comparing the thrust force required in the different helicopter flight regimes, we can say that the thrust required for flight in any regime is practically equal to the helicopter weight.

The power required for descent along an inclined trajectory consists of three parts: the motion power N_{mot}, the power required to create the lift force or inductive power N_i, and the power required to overcome the profile drag N_{pr}. If during descent the velocity and rpm are the same as in horizontal flight, the profile power is the same in both cases. The induced power is found from the formula $N_i = YV_i = G \cos \theta \, V_i$, and for descent angles up to 10° is practically equal to the induced power for horizontal flight, since in this case $\cos \theta \approx 1$.

The motion power for descent along an inclined trajectory is found from the formula

$$N_{mot} = P_x V. \tag{36}$$

From (34), $G_2 = X_{par} \pm P_x$. If the force P_x is directed along the helicopter motion, $P_x = X_{par} - G_2$. Then (36) can be written as

$$N_{mot} = (X_{par} - G_2)\, V = X_{par}V - G_2V.$$

The parasite drag forces are practically the same for descent and horizontal flight (for the same speed). Therefore

$$X_{par}V = N_{mot_h}.$$

We denote $G_2V = \Delta N$; then

$$N_{mot_{des}} = N_{mot_h} - \Delta N \tag{37}$$

i.e., the power required for descent is less than the power required for horizontal flight. Comparing (28), (32), (37), we can say that the most power is required for climbing flight and the least is required for descending flight. /111 The speed dependence of the power required for different flight regimes can be shown graphically with the aid of the Zhukovskiy grid, or by curves of the power required for various flight regimes, which are plotted for a given altitude (Figure 71a).

§ 56. Helicopter Rate of Descent With Operating Engine

During inclined descent the helicopter speed along the trajectory and the vertical rate of descent may vary from zero to the limiting permissible values. In accordance with the second descent condition, the speed along the trajectory will be constant if $G_2 = X_{par} \pm P_x$. The flight speed can be altered by varying the magnitude and direction of P_x. Since P_x depends on the position of the main rotor cone of rotation axis, there is a change of the helicopter speed along the trajectory when this position is changed. The helicopter motion velocity along the trajectory is connected with the vertical rate of descent as follows (Figure 70b)

$$V_{des} = V_{des_t} \sin \theta. \tag{38}$$

The vertical rate of descent is measured with the aid of a special instrument — a variometer. We see from the figure and Formula (38) that this rate depends on the velocity along the trajectory and the descent angle. The descent angle depends on the magnitude of the lift force Y, while the rate of descent depends on the force P_x, and therefore on the magnitude of the power supplied to the rotor. The helicopter rate of descent can be found from the Zhukovskiy grid.

If we draw on the Zhukovskiy grid, the lines showing power available N_{a_1}; N_{a_2}, for the given altitude, the points of their intersection with the /112 power required curves for the different regimes will correspond to definite

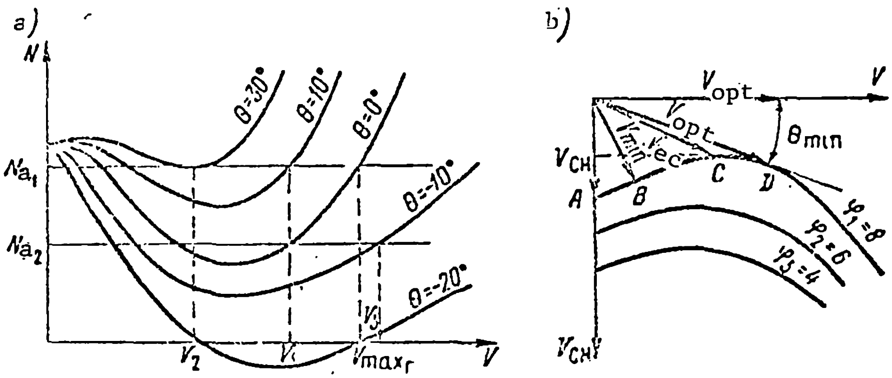

Figure 71. Power required versus flight speed for different climb and descent angles.

flight speeds along the trajectory. Thus, from the intersection of the line N_{a_1} with the power required curves we can find the maximal helicopter horizontal flight velocity V_{max}, the vertical rate of climb V_1 for $\Theta = 10°$, the vertical rate of climb V_2 for $\Theta = 30°$, and so on. Drawing the line N_{a_2}, we find the vertical rate of descent V_3 for $\Theta = -10°$. Other power available lines must be drawn, i.e., lower engine powers must be used, to find the rates of descent with larger descent angles.

The descent angles and velocities along the trajectory are often determined for some average altitude and some definite flight weight in the aerodynamic analysis. Such calculations are made for different values of main rotor pitch, for example, ϕ_1; ϕ_2; ϕ_3. From the results of this calculation we plot a graph — the polar curve of helicopter descent trajectories with operating engine (Figure 71b).

In this figure each curve corresponds to a definite velocity along the trajectory, if this velocity is plotted in the form of a vector from the coordinate origin. The horizontal axis will be the horizontal velocity

component $V_x = V_{des.t} \cos \theta$, and the ordinate axis will be the vertical rate of descent $V_{des} = V_{des.t} \sin \theta$. The vertical rate of descent is measured in m/sec, and the horizontal velocity is in km/hr. The velocity along the trajectory can be found if this velocity vector is transferred to the horizontal axis of the figure. Let us examine some characteristic points on the polar.

The point A lies at the intersection of the characteristic curve (for $\phi_1 = 8°$) with the ordinate axis. This point corresponds to helicopter vertical descent rate with the engine operating. If this rate of descent is more than 3 m/sec, transition to the vortex ring regime is possible and this cannot be permitted. This means that the pitch ϕ and the engine power must be increased.

The point B corresponds to descent with minimal permissible velocity along the trajectory, and the point C corresponds to descent with the minimal vertical rate of descent. In this case, the velocity along the trajectory corresponds to the economical velocity for horizontal flight of the helicopter. The point D corresponds to flight with the minimal descent angle θ_{min}, and in this case, the velocity along the trajectory will be equal to the optimal helicopter horizontal flight speed.

Programmed Testing Questions and Answers

Question 1. What is the lift force and how does it vary in the different flight regimes?

Answer 1. The lift force is the part of the thrust force developed by the main rotor which is directed perpendicular to the flight trajectory. In horizontal flight the lift force equals the helicopter weight $Y = G$. In climb and descent along an inclined trajectory the lift force will be less than the weight, i.e., $Y = G_1 = G \cos \theta$.

Answer 2. The helicopter lift force is the force equal to the main rotor thrust force. In horizontal flight the lift force $Y = G = T$; in climb this /113

force is greater than the helicopter weight; in an inclined descent it is less than the weight.

Answer 3. The lift force is the part of the main rotor thrust force which balances the normal component of the weight force. In horizontal flight the lift force equals the helicopter weight. In climb and inclined descent the lift force is less than the weight.

Question 2. Thrust required for various flight regimes and its variation in the different regimes.

Answer 1. The thrust required for a given regime is the force necessary to provide flight along the given trajectory at the required velocity. For constant velocity the thrust required is practically the same for horizontal flight, climbing flight, and descending flight.

Answer 2. The thrust required for a given flight regime is the force required necessary to balance the airplane weight and create the propulsive force. In a climb the thrust required will be considerably greater, and during descent it will be considerably less than in the horizontal flight regime.

Answer 3. The thrust required for a given regime is the force necessary to overcome the parasite drag. At a constant speed the thrust required will be the same for horizontal flight, inclined climb, and inclined descent.

Question 3. Power required for helicopter climb and descent along an inclined trajectory.

Answer 1. The power required for climb and descent consists of three parts: the power required to overcome profile drag, for horizontal motion of the helicopter, and for vertical motion of the helicopter.

During constant-velocity flight the profile power remains the same in all regimes. The power required for horizontal displacement also remains constant both for horizontal flight and for climb and descent. The power expended on vertical displacement will be more during climb, and less during descent. The sum of all these parts, i.e., the power for the given regime, will be practically the same in all regimes and varies with variation of the flight speed.

Answer 2. The power required for climb and descent along an inclined trajectory consists of three parts: the power required to overcome the profile drag; the power to create the lift force equal to the helicopter weight; and the power to overcome helicopter parasite drag.

During flight at constant speed the profile power remains constant in all regimes. The power required to create the lift force during climb will be more, and during descent will be less than in horizontal flight. If the velocity is unchanged, then the power required to overcome the parasite drag will remain unchanged. Consequently, the power required for climb is more than that for horizontal flight, while the power required for descent is less.

Answer 3. The power required for climb and descent along an inclined trajectory consists of three parts: the power required to overcome the profile drag, the power for creating the lift force, and that for motion of the helicopter along the given trajectory.

The profile power remains practically unchanged if the main rotor rpm remains the same in the various flight regimes. During climb and descent (climb and descent angles no more than 10°) the induced power remains unchanged and practically equal to the induced power for horizontal flight. The power required for motion during climb is more than the power required for motion in /114
horizontal flight by the magnitude of the excess power ΔN, i.e., $N_{cl} = N_h + \Delta N$, and the power required for motion during descent is less than the power required for horizontal flight by the amount ΔN.

Question 4. Vertical rate of climb and its variation with change of flight velocity and altitude.

Answer 1. The vertical rate of climb is the altitude which the helicopter gains per second. This rate depends on the excess power and the helicopter weight. The vertical rate of climb decreases with increase of the flight speed and latitude.

Answer 2. The vertical rate of climb is the altitude which the helicopter gains per unit time. The rate depends on the excess power and on the helicopter weight. The excess power increases as the flight velocity is increased from zero to the economical speed. Consequently, the vertical rate of climb will increase. The vertical rate of climb decreases with further increase of the velocity along the trajectory.

The vertical rate of climb increases with increase of flight altitude from zero to the engine critical altitude. The rate decreases at altitudes above the critical altitude.

Answer 3. The vertical rate of climb is the altitude which the helicopter gains per second. This rate will be the higher, the larger the excess power ΔN and the less the helicopter weight. The excess power decreases with increase of the flight speed; consequently, the vertical rate of climb also decreases. The power available and the excess power decrease with increase of the flight altitude.

This means that the higher the altitude, the lower the vertical rate of climb. The altitude at which the vertical climbing velocity equals zero is called the helicopter dynamic ceiling.

Question 5. Helicopter vertical rate of descent and what it depends on.

Answer 1. The vertical rate of descent is the altitude which the helicopter loses per second, i.e., $V_{des} = V_{des.t} \sin \theta$. It depends on the

velocity along the trajectory and the descent angle. The flight velocity along the trajectory depends on the main rotor thrust force component $\pm P_x$, directed parallel to the flight trajectory. The descent angle depends on the lift force Y, i.e., on the magnitude of the main rotor pitch.

The larger the main rotor pitch for the same rpm and the larger the backward tilt of the cone axis, the smaller the descent angle, velocity along the trajectory, and helicopter vertical rate of descent.

Answer 2. The vertical rate of descent is the altitude which the helicopter loses per second $V_{des} = V_{des.t} \sin \theta$. This rate depends on the velocity along the trajectory $V_{des.t}$ and the descent angle. The velocity along the trajectory will be the larger, the larger the propulsive force G_2, which is a part of the helicopter weight force ($G_2 = G \sin \theta$).

This means that the larger the descent angle, the larger the propulsive force G_2, the larger the flight velocity $V_{des.t}$, and the higher the vertical rate of descent.

Answer 3. The vertical rate of descent is the altitude which the helicopter loses per second ($V_{des} = V_{des.t} \sin \theta$). It is larger, the higher the velocity along the trajectory and the larger the descent angle. The vertical velocity along the trajectory depends on the propulsive force G_2 and the parasite drag force X_{par}. The larger the descent angle, the larger the propulsive force $G_2 = G \sin \theta$.

The larger the angle between the fuselage longitudinal axis and the flight trajectory, the larger the parasite drag force and the lower the velocity along the trajectory. This means that, by altering the position of the helicopter fuselage relative to the flight trajectory and by altering the descent angle, we can alter the flight velocity along the trajectory and the vertical rate of descent.

CHAPTER IX

HELICOPTER FLIGHT IN MAIN ROTOR AUTOROTATIVE REGIME

§ 57. Vertical Descent

So far we have examined helicopter flight with the engine operating. In /115
powered flight the thrust force developed by the main rotor performs the functions of lifting and propelling forces. But how is flight continued in case of engine failure?

In case of engine failure the helicopter can continue flight only in a descent (vertically downward or along an inclined trajectory). In this sort of flight the propelling force will be the weight force or its component parallel to the flight trajectory. The main rotor will turn, but the turning moment is supplied to the rotor by the aerodynamic forces acting on the rotor blades rather than from the engine. We shall first examine helicopter flight in the autorotative regime along a vertical trajectory (Figure 72).

During steady state vertical descent in the main rotor autorotative regime, the helicopter is acted on by the weight force G, main rotor thrust force T, drag force X of the nonlifting parts of the helicopter, and the tail rotor thrust force $T_{t.r}$. The helicopter travels vertically downward with the velocity V_{des}. The undisturbed flow approaches the helicopter from below at this same speed. As this flow passes through the area swept by the main rotor, it is subjected to the action of the blades. The blades of the rotating rotor tend to deflect the approaching stream downward. However, since the

vertical flow velocity V_{des} is greater than the induced velocity V_i which the main rotor blades create, the flow is only retarded rather than deflected downward. As a result of this retardation, the flow velocity V_1 above the rotor is less than the vertical descent velocity and is equal to the difference $V_1 = V_{des} - V_i$. Consequently, the mass of air flowing per unit time through the area swept by the rotor acquires a negative momentum increment $m_S V_i$, which in accordance with the law of momentum conservation will be equal to the main rotor thrust per unit time, i.e., $T = m_S V_i$, but $m_S = V_{des} F \rho$, then

$$T = FV_{des}\rho V_i. \tag{39}$$

Thus, the main rotor thrust in the autorotative regime will be larger, the larger the vertical rate of descent and the larger the flow retardation induced velocity. /116

We shall clarify the conditions for steady state descent in the autorotative regime on the basis of the diagram of the forces acting on the helicopter. These conditions are expressed by the equalities:

$$G = T + X;$$
$$T_{t.r} = S_s;$$
$$M_{cg} = 0.$$

The first condition ensures a constant rate of descent of the helicopter. The force X is the drag of all the nonlifting parts of the helicopter and acts in the direction of the thrust force; consequently, it retards the downward motion of the helicopter, and therefore, in this case, it cannot be termed the parasite drag force. The larger the force X, the lower the vertical rate of descent. But the drag force of the nonlifting parts is comparatively small and has no significant effect on the vertical rate of descent. Therefore, it can be neglected. Then the first condition is expressed by the approximate equality

$$G \approx T. \tag{40}$$

For this condition to be satisfied, it is necessary that the helicopter descend at a definite vertical velocity. Formula (39) shows that the thrust force equal to the helicopter weight can be obtained with a lower vertical velocity if the induced velocity is increased by increasing the main rotor pitch. But the main rotor pitch in the autorotative regime cannot be increased arbitrarily. Its magnitude must be strictly defined.

It has been established by experimental aerodynamics that the main rotor thrust in the autorotative regime is approximately equal to the total aerodynamic force R of a flat plate having an area equal to the area swept by the main rotor at an angle of attack of 90° (Figure 73).

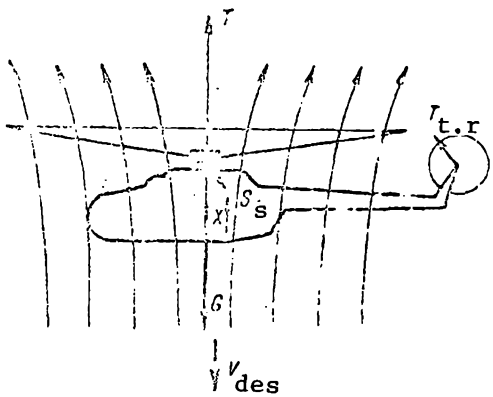

Figure 72. Vertical descent in autorotative regime.

Therefore, the rotor thrust must /117
be defined using the formula for the total aerodynamic force

$$T = R = C_R F \frac{\rho V_{des}^2}{2}, \qquad (41)$$

where C_R is the total aerodynamic force coefficient of a flat plate, equal to 1.2;

V_{des} is the helicopter vertical velocity.

Using (40) and (41), we find the helicopter vertical rate of descent

$$V_{des} = \sqrt{\frac{2G}{C_R F \rho}}.$$

Since G/F = P (specific loading per unit area swept by the rotor), the formula takes the form

$$V_{des} = \sqrt{\frac{2P}{C_R \rho}}. \qquad (42)$$

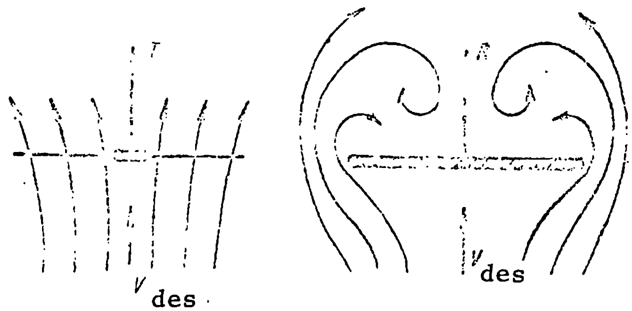

Figure 73. Thrust in autorotative regime.

We see from this formula that the helicopter vertical rate of descent depends on the specific loading on the area swept by the rotor (on the helicopter weight), and on the air density, and therefore on the flight altitude. The vertical rate of descent increases with increase of the specific loading (helicopter weight). This relation can be expressed by the formula

$$V_{des_2} = V_{des_1} \sqrt{\frac{G_2}{G_1}},$$

where V_{des_2} is the vertical velocity for helicopter weight G_2;

V_{des_1} is the vertical velocity for helicopter weight G_1.

With increase of the flight altitude, the air density decreases, which means that the vertical rate of descent increases and can be expressed by the equality

$$V_{des_H} = V_{des_0} \sqrt{\frac{1}{\Delta}}, \tag{43}$$

where V_{des_H} is the vertical velocity at the altitude H;

V_{des_0} is the vertical velocity at sea level;

Δ is the relative air density.

Formula (42) can be simplified if we consider $C_R = 1.2$; and $\rho_0 \frac{1}{8}$ kg/sec^2/m^4. Substituting these values into (42), we obtain

$$V_{des} = \sqrt{\frac{2P}{C_R \rho}} = \sqrt{\frac{2 \cdot 8P}{1.2}} \approx 3.6\sqrt{P}. \quad (44)$$

We find the vertical rate of descent of the Mi-1 helicopter in the autorotative regime if /118

$$G = 2200 \text{ kgf}; \quad F = 162 \text{ M}^2; \quad P = \frac{2200}{162} = 13.6 \text{ kgf/m}^2.$$

Then $V_{des} = 3.6\sqrt{13.6} = 13.3$ m/sec. This answer shows that a high vertical rate of descent is obtained even for a low specific loading. There are helicopters for which the specific loading on the main rotor reaches values as high as 25 kgf/m^2. For such helicopters, the vertical descent velocity is 18 m/sec or 65 km/hr.

We shall use the formula to find the vertical rate of descent of the Mi-1 helicopter at an altitude H = 3000 m, where $\Delta = 0.742$.

$$V_{des_H} = V_{des_0}\sqrt{\frac{1}{\Delta}} = 13.3\sqrt{\frac{1}{0.742}} = 15.5 \text{ m/sec.}$$

Thus, during vertical descent in the main rotor autorotative regime the helicopter travels with a high velocity, and landing is dangerous. The conditions for steady state flight in the autorotative regime differ fundamentally from the flight conditions with the engine operating. The difference is that there is no main rotor reactive moment in the autorotative regime. The blade aerodynamic forces do not retard rotor rotation; rather they create the turning moment. Therefore, the helicopter, in contrast with the case of flight with

the engine operating, will turn about the vertical axis in the direction of rotation of the main rotor. To eliminate this turning, the tail rotor must create a thrust directed oppositely to its thrust during flight with the engine operating. The main rotor side thrust force S_s will also be reversed in comparison with the side force during flight with the engine operating.

§ 58. Blade Aerodynamic Forces

In order to understand the essence of main rotor operation in the autorotative regime, it is necessary to examine the aerodynamic forces which arise on the blade element. In the autorotative regime, each blade element has two velocities: the circumferential velocity $u = \omega r$ and the vertical descent velocity V_{des} (Figure 74a). The sum of these velocities yields the resultant velocity $\overline{W} = \overline{u} + \overline{V}_{des}$. The air stream approaching the blade from the side has a direction opposite the blade element resultant velocity vector.

As a result of this flow pattern, the air pressure on the bottom of the blade will be higher than that above the blade. The total aerodynamic force ΔR is created on the blade element. This force may be directed forward at the angle γ relative to the main rotor hub rotation axis (Figure 74b); parallel to /119 the hub axis (Figure 74c); or back at the angle γ relative to the hub axis (Figure 74d). In the first case, the projection ($-\Delta Q$) of the force ΔR on the hub rotation plane will be directed along the rotation of the main rotor and forms a turning moment under the action of which the main rotor rpm will increase.

In the second case, the projection of the force ΔR on the hub rotation plane will be zero ($\Delta Q = 0$). Therefore, the force ΔR will have no retarding or accelerating effect on the rotation of the main rotor.

In the third case, the projection ΔQ of the force ΔR on the hub rotation plane will be directed aft, opposite the rotor rotation, and creates a retarding moment under the influence of which the main rotor rpm decreases. This

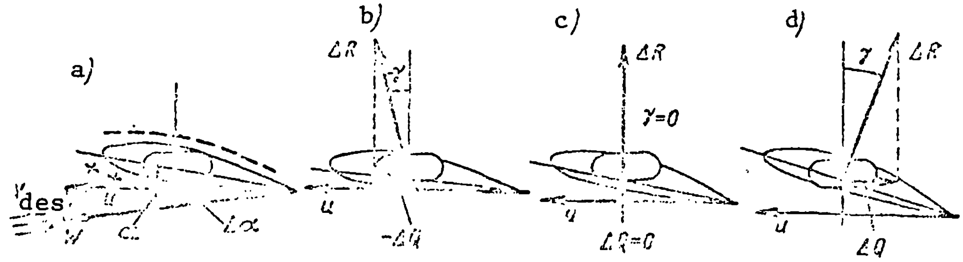

Figure 74. Tilt of force ΔR as a function of main rotor pitch.

means that the nature of the main rotor rotation is determined by the direction of the elemental forces which develop on the blade.

§ 59. Main Rotor Autorotation Conditions and Regimes

The main rotor blade element may be compared with an airplane wing element. Let us examine the aerodynamic forces acting on an airplane wing and then transfer these forces to the blade. The flight force Y and the drag force X develop on the airplane wing at the angle of attack α. The geometric sum of these forces will be the resultant aerodynamic force $\overline{R} = \overline{Y} + \overline{X}$. The angle between the lift force Y and the resultant aerodynamic force R vectors is called the aerodynamic efficiency angle (θ_K). The larger the aerodynamic efficiency angle, the lower the aerodynamic efficiency of the wing, since the minimal aerodynamic efficiency angle corresponds to maximal wing aerodynamic efficiency ctg θ_K = = Y/X = K. Reduction of the aerodynamic efficiency angle means a sort of "attraction" of the resultant aerodynamic force vector ΔR to the lift force vector Y, i.e., it means reduction of the backward tilt of ΔR relative to the normal to the undisturbed flow.

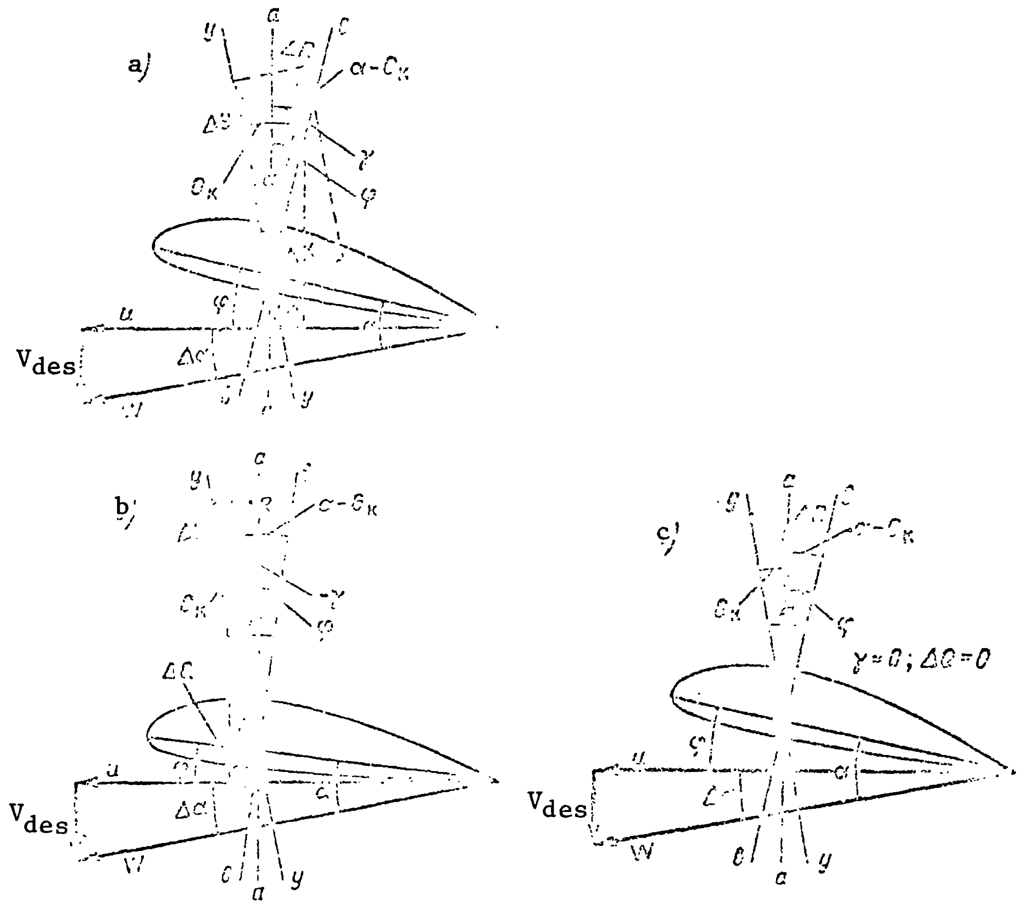

Figure 75. Blade element autorotative conditions.

Now let us turn to examination of the forces acting on the blade. We draw three straight lines through the center of pressure of the blade element: a-a is perpendicular to the rotor rotation plane; b-b is perpendicular to the blade element chord; y-y is perpendicular to the resultant velocity vector (Figure 75a). The angle between the lines y-y and b-b and the angle α are equal, since they are formed by mutually perpendicular sides. On this same basis, the angle between the lines a-a and b-b is equal to the incidence angle ϕ.

The resultant aerodynamic force vector is applied at the blade element center of pressure. We resolve this force into the lift force ΔY and the drag force ΔX. The angle between the vectors ΔY and ΔR is the aerodynamic efficiency angle θ_K. The angle between the vector ΔR and the line b-b will be equal to the difference of the angles $(\alpha - \theta_K)$. If this difference is less than the blade incidence angle, then $\phi - (\alpha - \theta_K) = \gamma$, i.e., the angle γ is positive. This means that the projection ΔQ of the force ΔR on the main rotor plane of rotation will be directed aft and creates a retarding moment which reduces the rotor rpm.

The main rotor will operate in the decelerated autorotative regime, which leads to stopping of the rotor. The larger the blade element incidence angle or pitch, the larger the angle γ, and the larger the force ΔQ and its decelerating moment.

If the difference between the angle of attack and the aerodynamic efficiency angle is greater than the incidence angle, i.e., $(\alpha - \theta_K) > \phi$, then $\phi - (\alpha - \theta_K) = -\gamma$. The angle γ is negative, which means that the vector ΔR /121 is directed forward relative to the hub axis (Figure 75b). The projection ΔQ of the force ΔR on the hub rotation plane is directed forward and creates a turning moment which accelerates the rotor rotation. The main rotor will operate in the accelerated autorotative regime. The smaller the incidence angle ϕ, the larger the forward tilt of the vector ΔR, and the higher the speed at which the rotor turns.

If the difference $(\alpha - \theta_K)$, then $\phi - (\alpha - \theta_K) = 0$, i.e., the force ΔR is parallel to the hub axis and its projection on the hub rotation plane $\Delta Q = 0$ (Figure 75c). In this case, the retarding or turning moment equals zero, and the rotor revolves at constant rpm, i.e., the rotor autorotative regime will be established.

From these examples we conclude:

— the tilt of the elemental resultant aerodynamic force vector depends on the blade element pitch;

— with reduction of the pitch, the force vector ΔR is deflected forward, and the main rotor autorotation becomes accelerated;

— with increase of the blade element pitch, the force vector ΔR is deflected aft, and the main rotor autorotation becomes decelerated.

The dependence of the autorotative regime on the blade element angle of attack and pitch can be expressed graphically (Figure 76). This graph is called the autorotation margin graph. The abscissa is the blade element angle of attack, the ordinates are the incidence angles ϕ and the angles equal to the difference $\alpha - \theta_K$.

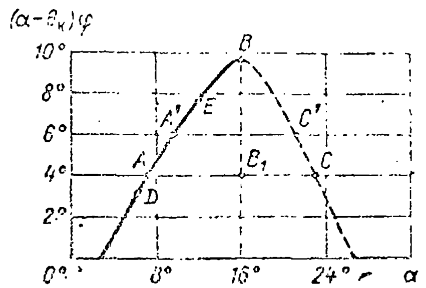

Figure 76. Mi-1 helicopter blade autorotation margin.

Let us examine the characteristic points in this figure. The ascending portion of this curve corresponds to blade element angles of attack below stall. The point B is the stall angle, and the descending portion corresponds to angles above stall.

If we draw a straight line parallel to the abscissa axis, it crosses the curve at the two points A and C. In the figure, this straight line passes through the point on the ordinate corresponding to the incidence angle $\phi = 4°$ (such a straight line can be drawn through any point of the ordinate). What do the points A and C characterize? The point A corresponds to the blade element angle of attack (in our example $\alpha = 7°30'$) which corresponds to steady state autorotation. Let us show that this actually is the case.

It was established above that steady state autorotation will occur when the difference $\alpha - \theta_K = \phi$. In this case, the force ΔR of the element will be parallel to the main rotor hub axis. In our example $\phi = 4°$ and $\alpha - \theta_K = 4°$.

This means that $\gamma = 0$ (see Figure 75c). Therefore, in order for the Mi-1 helicopter blade to have steady state autorotation with $\phi = 4°$ the blade angle of attack must equal 7° 30'. Points of the curve for which $\alpha - \theta_K < 4$ correspond to smaller angles of attack, i.e., the autorotation will be decelerated (see Figure 75a). At angles of attack between the points A and C all the points of the curve correspond to the inequality $\alpha - \theta_K > \phi$, i.e., accelerated autorotation (see Figure 75b). But at angles of attack up to the stall angle, the accelerated autorotation will be stable, while at angles above stall flow separation takes place, and the autorotation is unstable. /122

The range of angles of attack between the points A and C is called the blade element autorotation margin. Since flight with blade element angles of attack above the stall angle is not feasible in practice, the autorotation margin will correspond to the angles of attack between the points A and B_1 in the figure.

With increase of the blade element pitch, the straight line AC shifts upward (A'C'). This means that the angle of attack range corresponding to decelerated autorotation increases, and the autorotation margin decreases. With reduction of the pitch, the straight line AC shifts downward, and the autorotation margin increases.

Since the angles of attack are different for different blade elements, the autorotation conditions for these elements will also be different, and therefore the autorotation margin graph has a somewhat arbitrary nature, i.e., it serves for a qualitative evaluation of this process.

Let us return to the point A on the autorotation margin graph. In our example, it corresponds to $\phi = 4°$ and $\alpha = 7°30'$. At these angles, the autorotation will be steady-state. But how can we obtain an angle of attack $\alpha = 7°30'$ with an incidence angle $\phi = 4°$?

Since $\alpha = \phi + \Delta\alpha$ (see Figure 75b), then $\Delta\alpha = \alpha - 4$. This means that for our example the angle of attack increment caused by the vertical rate of

descent is $\Delta\alpha=7°30'-4°=3°30'$. Let us find the vertical rate of descent corresponding to this $\Delta\alpha$ if $\bar{r}=0.7$; $\phi=4°$; $\omega=26$ rad/sec. It is known that

$$\mathrm{tg}\ \Delta\alpha = \frac{V_{des}}{\omega r}$$

hence

$$V_{des} = \omega r\ \ \mathrm{tg}\Delta\alpha$$

or

$$V_{des} = \omega\bar{r}R\ \Delta\alpha = 26\cdot 0.7\cdot 7.17\cdot 0.061 = 7.9\ \mathrm{m/sec}.$$

Thus, for $V_{des} = 7.9$ m/sec steady-state autorotation will take place.

If the vertical velocity $V_{des} < 7.9$ m/sec, then for the given blade element the angle of attack becomes less than 7°30', and the autorotation will be decelerated;conversely, if $V_{des} > 7.9$ m/sec, the autorotation will be accelerated. We must emphasize once again that the words "accelerated auto- /123 rotation of the element" are arbitrary. They mean that under the given conditions the elemental resultant aerodynamic force vector ΔR is inclined forward relative to the hub axis and creates a turning moment. The larger the helicopter vertical rate of descent, the larger the angle of attack increment $\Delta\alpha$, the larger the forward tilt of the force vector ΔR, and the higher the main rotor rpm will be.

§ 60. Conditions for Autorotation of Different Blade Elements

We have examined above the conditions for rotor autorotation as a function of blade element pitch and angle of attack. To facilitate our understanding of the problem,we assumed that all the blade elements operate under the same conditions, i.e., all the elements have the same incidence angles, velocities, the same forces ΔR, and the same inclination of these forces. But in reality,

each blade element operates under different autorotation conditions. Let us examine these conditions.

The angle of attack increment $\Delta\alpha$ depends not only on the vertical rate of descent, but also on the circumferential velocity of the blade element. The circumferential velocity is considerably higher for the tip elements than for the root elements; therefore, the angle of attack increment of the tip element is less than that of the root element, i.e., $\Delta\alpha_1 < \Delta\alpha_2$ (Figure 77a). But then the elemental force vectors ΔR for the tip elements are inclined aft because of the low value of $\Delta\alpha$ and create retarding moments. The blade tip elements usually operate in the decelerated autorotation regime. The retarding action of the tip elements is reduced by geometric twist of the blade but is not eliminated entirely.

The value of $\Delta\alpha$ will be large for the root elements; therefore, the elemental force vectors ΔR will be tilted forward, and their projections ΔQ provide a turning moment. Consequently, the blade root elements operate under accelerated autorotation conditions. Since the blade tip elements retard the rotation while the root elements accelerate the rotation, what will be the operating regime of the entire rotor?

With reduction of the rotor pitch and with increase of the vertical rate of descent, the retarding action of the tip elements is less than the accelerating action of the root elements. The resultant of the elementary forces is directed forward, in the direction of rotor rotation and forms a turning moment (Figure 77b). In this case, the main rotor autorotation regime will be accelerated.

With increase of the pitch or reduction of the vertical rate of descent, the retarding action of the blade tip elements increases. If in this case the resultant Q_b of the elemental forces is zero, the rotor autorotation regime will be steady-state (Figure 77c). If the retarding action of the tip elements /124
exceeds the accelerating action of the root elements, the resultant Q_b of the

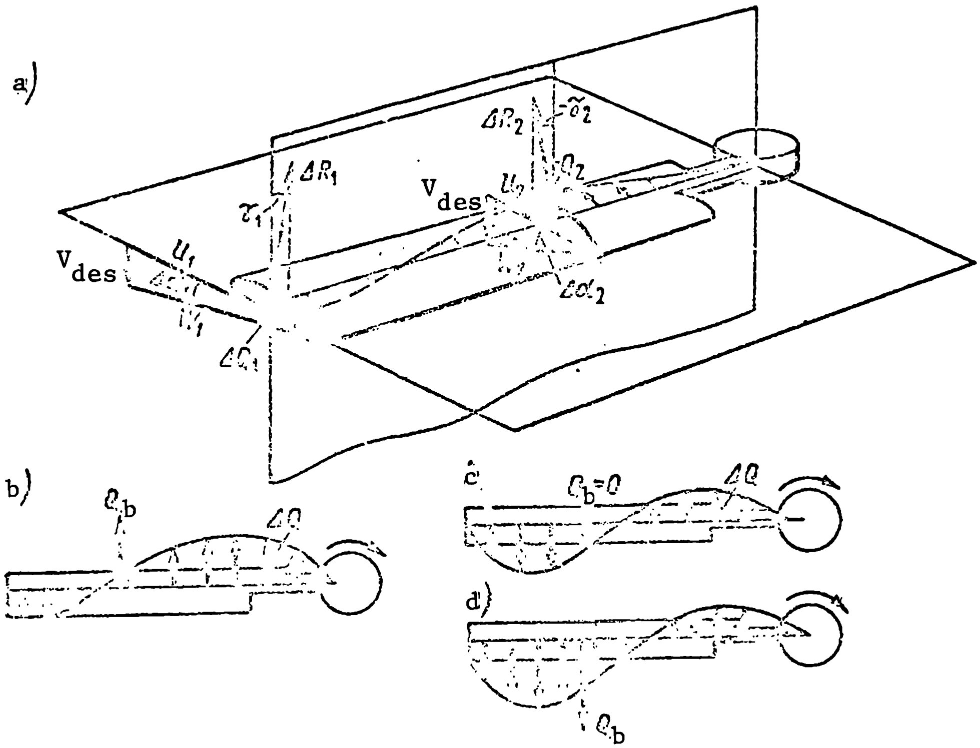

Figure 77. Autorotative conditions of different blade elements.

elementary forces is directed aft (Figure 77d) and creates a retarding moment. The rotor will operate in the decelerated autorotation regime.

Let us confirm these conclusions by an example.

Let $\bar{r}_1 = 0.98$; $\bar{r}_2 = 0.42$; $\bar{r}_3 = 0.28$.

The rotor blade twist $\Delta\phi=4°$ is provided between the relative radii $\bar{r} = 0.3 - 0.5$. The main rotor pitch is defined by the pitch of the blade

element with the relative radius $\bar{r} = 0.7$.

The main rotor pitch for our example is $\phi = 4°$. Then

$$\varphi_1 = 4°;\quad \varphi_2 = 6°;\quad \varphi_3 = 8°.$$

The main rotor radius of the Mi-1 helicopter is R = 7.17 m. Then

$$r_1 = 0.98 \cdot 7.17 = 7 \text{ m};\quad r_2 = 0.42 \cdot 7.17 = 3 \text{ m};$$
$$r_3 = 0.28 \cdot 7.17 = 2 \text{ m}.$$

Let us find the circumferential velocities of the selected blade elements if

$$\omega = 26 \text{ rad/sec};\ V_{des} = 8 \text{ m/sec}.$$

We find the angle of attack increments /125

$$\Delta\alpha_1 = \operatorname{arctg}\frac{8}{182} = 2°30';$$

$$\Delta\alpha_2 = \operatorname{arctg}\frac{8}{78} = 6°;$$

$$\Delta\alpha_3 = \operatorname{arctg}\frac{8}{52} = 8°45'.$$

We calculate the angles of attack of the elements

$$\alpha_1 = \varphi_1 + \Delta\alpha_1 = 4° + 2°30' = 6°30'.$$
$$\alpha_2 = 6° + 6° = 12° \text{ and}$$
$$\alpha_3 = 8° + 8°45' = 16°45'.$$

Using the autorotation margin graph (see Figure 76), we find the autorotative regimes of the given blade elements. To this end we take three straight lines parallel to the abscissa axis, drawn through the ordinate points corresponding to the incidence angles $\varphi_1 = 4°$; $\varphi_2 = 6°$; $\varphi_3 = 8°$. The

point D in the figure corresponds to the blade tip element with the angle of attack $\alpha_1 = 6°30'$. This point lies in the decelerated autorotation regime $(\alpha - \theta_K = 3°15')$. For this blade element the resultant aerodynamic force ΔR is tilted aft through the angle $\gamma = \varphi - (\alpha - \theta_K) = 4° - 3°15' = 0°45'$.

The point E, for which $\alpha - \theta_K = 7°15'$, corresponds to the second blade element with the angle of attack $\alpha_2 = 12°$. The pitch of this element is $\varphi_2 = 6°$. This means that this element is in the accelerated autorotation regime, and its aerodynamic force ΔR is tilted forward by the angle $-\gamma = \varphi - (\alpha - \theta_K) = 6° - 7°15' = -1°15'$ The point B, for which $\alpha - \theta_K =$ [illegible], corresponds to the third blade element (angle of attack $\alpha_3 =$ [illegible]). The pitch of this element is $\varphi_3 = 8°$. This element also is in the accelerated autorotation regime, but its angle of attack is close to the stall angle, and the resultant aerodynamic force vector ΔR is tilted forward through the angle $-\gamma = 8° - 9°30' = -$ [illegible].

The blade elements located closer to the hub axis will have angles of attack above the stall angle, i.e., they will operate under separated flow conditions. In our example, most of the blade elements operate under accelerated autorotation conditions, which means that the main rotor will operate in the accelerated autorotative regime.

In our example ω = 26 rad/sec or n = 250 rpm. For this rotor this rpm will be maximal, and further increase of the rpm is not permissible. The main rotor pitch must be increased to obtain the steady-state autorotative regime. It is left to the reader to calculate for himself the approximate value of the pitch corresponding to this regime. The following comment must be added to what we have said. During flight in the autorotative regime, the helicopter will turn in the direction of rotation of the main rotor as a result of transmission shaft friction torque. In order to eliminate this turning, a thrust moment of the tail rotor, which turns under the influence of main rotor torque, must be created. Consequently, for steady-state autorotation the blades must create a small torque, which overcomes the friction torque in the transmission and the reactive torque of the tail rotor. We recall that the tail rotor creates thrust opposite in direction to the force which is generated in flight /126

with the engine operating (i.e., the tail rotor operates at small negative incidence angles; therefore, comparatively little driving torque is required).

§ 61. Gliding

Rectilinear flight of the helicopter along an inclined trajectory with the main rotor operating in the autorotative regime is termed gliding (Figure 78). In this flight regime the helicopter is subject to the forces: helicopter weight G, main rotor thrust T, parasite drag X_{par}, and tail rotor thrust $T_{t.r}$.

We resolve the helicopter weight force into two components: G_2 directed along the flight trajectory, and G_1 perpendicular to the trajectory.

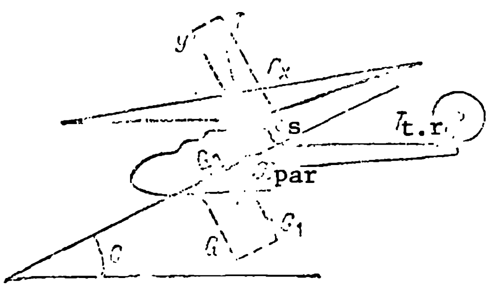

Figure 78. Forces acting on helicopter in glide.

We resolve the main rotor thrust force into the lift force Y and the drag force P_x.

The steady-state gliding conditions will be expressed by the equalities:

$$Y = G_1 = G \cos \theta;$$

$$G_2 = X_{par} + P_x = G \sin \theta;$$

$$T_{t.r} = S_s;$$

$$\sum M_{c.g} = 0.$$

The first condition ensures constant gliding angle and rectilinear flight; the second assures constant speed along the trajectory. The tail rotor thrust is directed in the opposite direction in comparison with the thrust in the descent regime with the engine operating. The velocity along the trajectory and the gliding angle can be altered by tilting the thrust

force vector forward or aft, and also by varying the main rotor pitch. But we recall that flight takes place in the autorotative regime, and therefore the /127 pitch can be altered only within the permissible rpm limits. The pitch cannot be increased markedly, since the rotor may transition into the decelerated autorotation regime, and the rpm may become less than the minimal permissible value.

The main rotor autorotation conditions in a glide are much more complex than in a vertical descent. This is basically the result of two factors: azimuthal variation of the flow velocity over each blade element and the presence of blade falpping motions caused by transverse flow over the main rotor.

In a vertical descent each blade element has a constant velocity $W = \sqrt{u^2 + V_{des}^2}$. In a glide this velocity depends on the blade azimuth and changes continuously. In a vertical descent we can assume the absence of flapping motions, which simplifies the analysis of the blade element autorotative conditions. The flapping motions must be considered in a glide. But the derivations of the autorotation conditions which were carried out for the vertical descent remain valid for the gliding conditions as well. Let us recall these conclusions.

The autorotation conditions depend on the blade element pitch and the pitch of the entire main rotor: the lower the pitch, the greater the forward tilt of the force vector ΔR and the higher the main rotor rpm.

The larger the angle of attack increment caused by the vertical descent velocity, the larger the forward tilt of the force vector ΔR and the higher the main rotor rpm.

The latter conclusion is particularly important in clarifying the autorotation conditions in a glide, therefore, we shall examine the diagram in Figure 79. We see from the figure that the angle $\Delta\alpha$ is equal to the angle between the lines y-y and a-a (y-y is perpendicular to the resultant velocity vector, a-a is perpendicular to the plane of rotation, or parallel to the hub rotation axis). Consequently, if $\Delta\alpha > \theta_K$, the force vector ΔR will be tilted forward relative to the hub axis by the angle $-\gamma$, and the rotor autorotation will be accelerated. The larger $\Delta\alpha$, the higher the main rotor rpm.

Keeping this analysis in mind, we turn to examination of the autorotation characteristics in a glide. We first examine the influence on the autorotation conditions of the azimuthal variation of the resultant blade element velocity. Since the resultant /128
velocity varies continuously in azimuth, we cannot analyze this variation directly. Therefore, we take the two most characteristic azimuths, 90 and 270°, and we compare the autorotation conditions at these azimuths (Figure 80). The flight direction along the trajectory is shown in the figure by the arrow DF. The main rotor hub rotation plane is horizontal, the angle of attack of the main rotor lies between this plane and the gliding velocity vector and equals the gliding angle A = θ. At the 90° azimuth in the hub rotation plane, the flow approaches the blade element with the velocity $\omega r + V \cos\theta$ (Figure 80a). The blade element angle of attack increment caused by the vertical descent velocity can be found from the formula

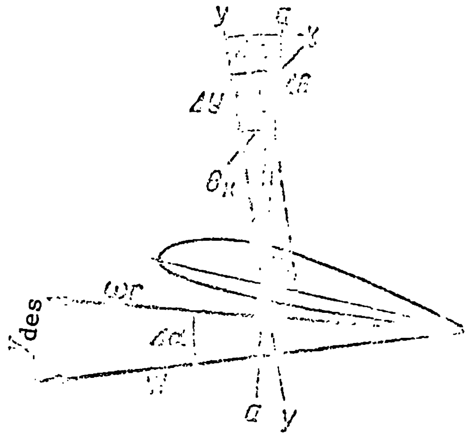

Figure 79. Dependence of autorotative conditions on vertical rate of descent.

$$\mathrm{tg}\,\Delta\alpha = \frac{V_{gl} \sin\theta}{\omega r + V_{gl} \cos\theta}. \tag{43}$$

At the 270° azimuth the blade travels aft relative to the direction of flight, therefore, the flow approaches the blade element in the hub rotation plane with the velocity $\omega r - V_{gl} \cos\theta$ (Figure 80b). In this case, the angle of attack increment is

$$\mathrm{tg}\,\Delta\alpha = \frac{V_{gl} \sin\theta}{\omega r - V_{gl} \cos\theta}. \tag{44}$$

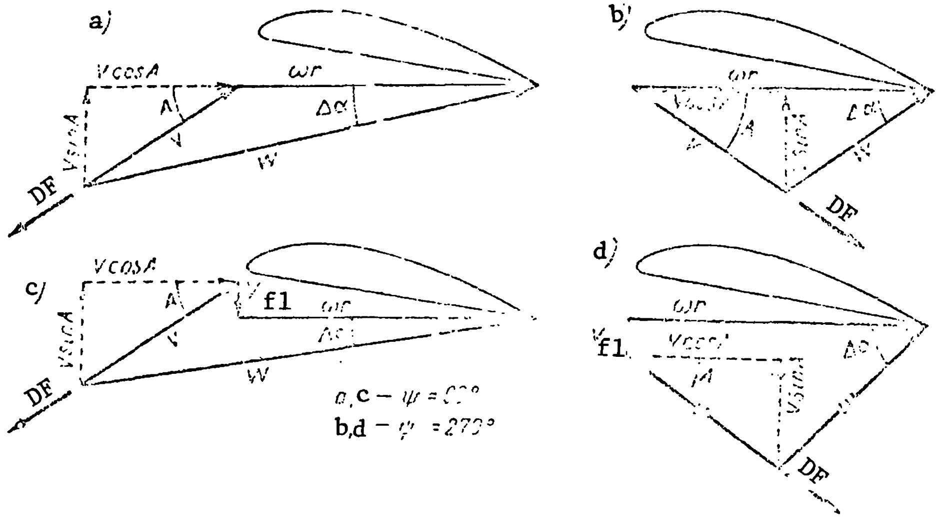

Figure 80. Dependence of autorotative conditions on azimuth and flapping motions:

a, c – ψ = 90°; b, d – ψ = 270°

Comparing diagrams a and b, and also Formulas (43) and (44), we conclude that the angle of attack increment at the 270° azimuth is larger than at the 90° azimuth. Consequently, as a result of the variation of the resultant velocity in the gliding regime, there is a change of the angle of attack increment and tilt of the resultant aerodynamic force: at the 90° azimuth the tilt is aft; at the 270° azimuth the tilt is forward.

Now let us examine the influence of flapping motions on the autorotation /129
conditions.

As the advancing blade flaps up, the angles α and $\Delta\alpha$ decrease. At the 90° azimuth, where the upward vertical flapping velocity reaches its maximal value (Figure 80c), the angle of attack increment becomes minimal

$$\operatorname{tg} \Delta\alpha = \frac{V_{gl} \sin\theta - v_{fl}}{\omega r + V_{gl} \cos\theta} .$$

Therefore, the maximal aft tilt of the force vector ΔR and the maximal blade retarding moment occur at this azimuth. The maximal down vertical flapping velocity will occur at the 270° azimuth. Therefore, in accordance with the formula

$$\operatorname{tg} \Delta\alpha = \frac{V_{gl} \sin\theta + v_{fl}}{\omega r - V_{gl} \cos\theta}$$

the blade element has the largest angle of attack increment (Figure 80d). The maximal forward tilt of the elemental force and the maximal turning moment will occur at this azimuth.

Thus, we draw the following conclusion. During gliding, the autorotation conditions of each blade element and the entire blade as a whole vary during a single revolution of the rotor. The advancing blade creates a retarding moment, which reduces the rotor rpm. The maximal retarding moment is created at the $\psi = 90°$ azimuth. The retreating blade creates a turning moment whose maximal value occurs at the 270° azimuth, where the angle of attack increment $\Delta\alpha$ becomes maximal. This means that during a glide, the blades alternately accelerate and retard the rotation, and on the whole, the main rotor operates under steady-state autorotation conditions. The rotor rpm is regulated by the pitch: the lower the pitch, the higher the rotor rpm.

§ 62. Vertical Rate of Descent in a Glide

The vertical rate of descent is the altitude through which the helicopter descends per second (Figure 81). This rate is found from the formula

$$V_{des} = V_{gl} \sin\theta.$$

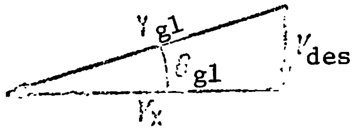

Figure 81. Rate of descent as a function of gliding speed and angle.

The vertical velocity will be constant for constant glide angle and constant velocity along the trajectory. This means that the propulsive force in a glide is the weight force component /130 $G_2 = G \sin \theta$. The work of this force per unit time will be power, equivalent to the power required for horizontal flight at a velocity equal to the gliding velocity. Consequently,

$$G_2 V_{gl} = N_h = G \sin \theta \; V_{gl}$$

or

$$N_h = \frac{GV_{des}}{75}. \qquad (45)$$

This means that the work per unit time of the helicopter weight force will be equivalent to the power supplied to the main rotor shaft in horizontal flight at the same velocity with which the helicopter glides in the autorotative regime. From (45) we can find the vertical rate of descent during gliding

$$V_{des} = \frac{75N_h}{G} \qquad (46)$$

where N_h is the power required for horizontal flight, hp;
G is the helicopter weight, kgf.

We see from (46) that the minimal vertical rate of descent will be achieved when gliding at the economical speed, since V_{ec} corresponds to the required power N_{min}, i.e.

$$V_{des_{min}} = \frac{75N_{min}}{G}.$$

For rough calculations we can use the approximate formula $N_{min} = 1/2 N_{rat}$, but then we must introduce the power utilization coefficient ζ, since $N_h = N_e \zeta$ Taking the value of this coefficient to be $\zeta = 0.8$, we find

$$V_{des_{min}} = \frac{75 \cdot 0.8 \cdot 0.5 N_{rat}}{G} = \frac{30 N_{rat}}{G}. \tag{47}$$

We note that the ratio $G/N = q$ is called the power loading, and the equality takes the form

$$V_{des_{min}} = \frac{30}{q_{rat}} \tag{48}$$

$q_{rat}\sqrt{P} = E_M$ is the helicopter energy efficiency;

P is the disk loading.

Then $q_{rat} = \frac{E_M}{\sqrt{P}}$

We substitute this value into (48)

$$V_{des_{min}} = \frac{30\sqrt{P}}{E_M}.$$

The average value of the energy efficiency is $E_M \approx 20$. Then the formula for determining the minimal helicopter gliding vertical rate of descent will be

$$V_{des_{min}} = \frac{30}{20}\sqrt{P} = 1.5\sqrt{P}.$$

We recall that this formula is approximate, but it yields adequately precise results, although somewhat low.

Comparison of the vertical rates of descent in glide and vertical descent makes it possible to say that the vertical velocity in the glide will be

2-2.5 times less than in the vertical descent. Therefore, gliding is used in all cases if surrounding obstacles do not interfere.

However, gliding can be performed with other velocities along the trajectory rather than the economical velocity. A special graph — the helicopter glide trajectory polar curve — is constructed for determining the gliding velocities and angles. We use the power required and available curves for horizontal flight to construct this graph. If we draw on these curves a straight line N_1 parallel to the horizontal axis, it crosses the power required curve at the points A and B (Figure 82a). The point A corresponds to the horizontal flight velocity V_1, and the point B corresponds to the velocity V_2.

We take these velocities as the horizontal components V_x of the gliding velocity (Figure 82b): $V_1 = V_{x_1}$; $V_2 = V_{x_2}$. Since these velocities correspond to one and the same power required N_1 for horizontal flight, we use (46) to find the vertical descent velocity. It will be the same for gliding with the velocities V_{x_1} and V_{x_2}. After determining V_{des} and V_x, we find the gliding velocity

$$V_{gl_1} = \sqrt{V_{x_1}^2 + V_{des}^2}; \quad V_{gl_2} = \sqrt{V_{x_2}^2 + V_{des}^2}.$$

Then we find the gliding angle

$$\theta = \operatorname{arctg}\frac{V_{des}}{V_x}, \text{ or } \theta = \arcsin\frac{V_{des}}{V_{gl}}.$$

Conclusion: the same vertical rate of descent corresponds to two gliding regimes with large and small gliding angle, with high and low velocity along the trajectory. We usually select gliding with the lower angle but the higher velocity along the trajectory. After making these calculations for several vertical velocities, we plot the helicopter glide polar (Figure 83). From this plot we can find: /132

— the minimal vertical rate of descent;

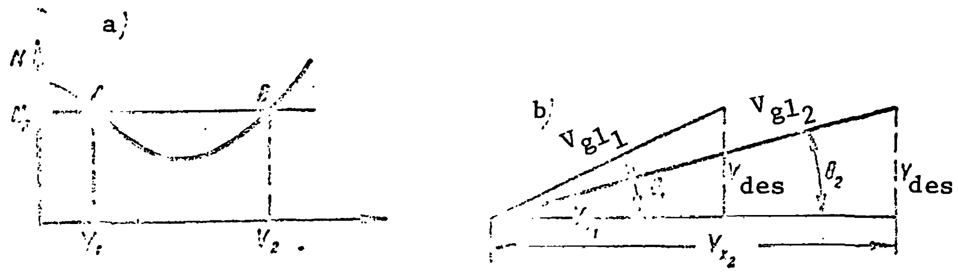

Figure 82. Vertical rate of descent versus gliding angle.

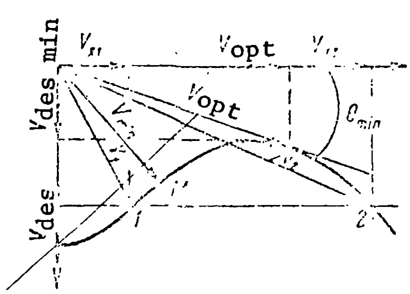

Figure 83. Gliding trajectory characteristic curve.

— the minimal gliding angle which corresponds to the velocity V_x, numerically equal to the optimal horizontal flight speed;

— the vertical velocity in a vertical descent (point of intersection of the polar curve with the vertical axis).

If we draw through the polar a secant parallel to the horizontal axis, the points of intersection 1 and 2 will correspond to two gliding regimes, for which the vertical rate of descent will be the same and the velocities along the trajectory will be different. The point M on the polar corresponds to the minimal velocity along the trajectory.

§ 63. Safety Height

The minimal vertical rate of descent at which flight is safest is achieved when gliding at the economical speed. However, if the engine fails while hovering, the helicopter speed V = 0. In order to transition into a glide at a speed close to the economical speed, some altitude must be lost in

order for the helicopter to acquire a definite kinetic energy $E = \frac{mV^2_{gl}}{2} = \frac{GV^2_{gl}}{2g}$. Only part rather than all of the helicopter's potential energy is used in acquiring the velocity (approximately two tenths of the total potential energy). The remaining energy goes to overcome parasite drag and main rotor profile drag, to turn the tail rotor and the accessories. The total potential energy of the helicopter is

$$E_b = GH,$$

where G is the helicopter weight;

H is the helicopter flight altitude.

We find the kinetic energy with loss of altitude from the formula

$$0.2GH = \frac{GV^2_{gl}}{2g}.$$

Hence, we find the safe helicopter hover height /133

$$H = \frac{V^2_{gl}}{0.2 \cdot 2g} \approx \frac{V^2_{gl}}{4}.$$

But experience shows that an additional height margin of about ten meters is required for the landing maneuver; therefore, the formula for determining the safe hover height takes the form

$$H_{saf} = \frac{V^2_{gl}}{4} + 10.$$

Example. The economical flight speed for the Mi-1 helicopter is V_{ec} = 90 km/hr or 24.8 m/sec. We find the safe hovering height.

$$H_{saf} = \frac{24.8^2}{4} + 10 = 165 \text{ m}.$$

If the helicopter has translational velocity in the horizontal direction prior to transition to the main rotor autorotative regime, the safe height is found from the formula

$$H_{saf} = \frac{V^2_{gl} - V^2_h}{4} + 10.$$

For example, the Mi-1 helicopter is flying horizontally at a speed of 70 km/hr or 20 m/sec. In this case, the safe height is defined as

$$H_{saf} = \frac{25^2 - 20^2}{4} + 10 = 66 \text{ m}.$$

Therefore, in case of engine failure in horizontal flight or in climb along an inclined trajectory, less altitude is required for transition into the autorotative regime than for transition into this regime from hover or when performing vertical climb or vertical descent with the engine operating. After determining the safe heights for transition into the autorotation regime for different flight speeds, we can plot the safe flight height diagram (Figure 84).

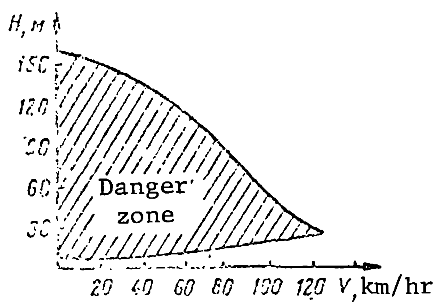

Figure 84. Helicopter flight danger zone.

This diagram shows the danger zone, and we see that the safe hover height is up to ten meters or above 200 meters. The safe hover and flight height limitation at low speeds makes the use of helicopters at low altitudes difficult in practice. It is not advisable to fly the helicopter in the danger zone except in extreme emergencies.

§ 64. Transition From Flight With Engine Operating to Flight in the Main Rotor Autorotation Regime /134

Gliding in the autorotative regime is not an emergency flight mode; rather it is a normal, stable flight mode which is often used even with a normally functioning, sound engine. Gliding in the autorotative regime is

used for working out basic piloting techniques or for rapid loss of altitude. However, in order to assure safety, a definite order and sequence of actions must be followed in transitioning from flight with the engine operating into the autorotative regime. What happens with the helicopter in the case of sudden engine stoppage or in case of rapid decrease of the engine rpm?

The main rotor continues to rotate momentarily, and the rpm does not change, since the freewheeling clutch automatically disengages the engine from the transmission. The rotor continues to turn by inertia, since it has definite angular momentum. The greater the mass or weight of the blades, the larger the moment of inertia, the longer time the rotation will continue by inertia. Therefore, heavy blades have an advantage in the autorotative regime.

Under the action of the reactive moment, the main rotor rpm decreases, and therefore the thrust decreases. If the engine fails in the hovering regime, then as a result of main rotor thrust reduction, the helicopter will transition to vertical descent. However, if the engine fails in horizontal flight, reduction of the thrust and lift will cause the helicopter to descend along an inclined trajectory. In both cases, the air flow will approach the main rotor from below.

The presence of the vertical velocity causes increase of the blade element angles of attack by the magnitude $\Delta\alpha$ and deflection of the force vector ΔR forward, i.e., a driving torque appears; therefore, there is an increase of the rpm or at least no further reduction of the rpm. Moreover, along with reduction of the main rotor rpm, there is reduction of the centrifugal force of each blade, which leads to increase of the main rotor coning angle, i.e., simultaneous upward flapping of the blades. When the flapping angle increases, there is a reduction of blade pitch under the influence of the flapping compensator, i.e., there is an increase of the main rotor rpm.

Thus, we conclude that in the case of engine failure, there are objective factors which facilitate transition of the main rotor into the autorotative regime. But the pilot must not rely on these conditions and expect the rotor

itself to transition into autorotation. Therefore, in case of engine failure, /135
the pilot must immediately reduce main rotor pitch to the minimal value. To this end, the collective-throttle lever is lowered fully. The main rotor rpm increases, and the circumferential velocity of the blade elements increases. This leads to reduction of the blade element angles of attack and aft deflection of the force ΔR. Therefore, the main rotor rpm will increase up to some limit, and then the constant rpm regime is established, i.e., the autorotation becomes steady. However, if the rpm is too high, the pitch must be increased somewhat. During flight with the engine not operating, the rpm should correspond to the engine rated power rpm. In this case, the rotor will develop the maximal thrust force, and the vertical rate of descent will be minimal.

Transition of the main rotor into the autorotative regime is facilitated by the stabilizer mounted on the tail boom. The stabilizer incidence angle changes with change of the main rotor pitch: when the pitch is reduced to the minimal value, the stabilizer incidence angle becomes negative (Figure 85). If at the time of transition into the autorotative regime the helicopter is moving with a horizontal velocity, the negative lift force Y_{st} develops on the stabilizer. The moment of this force $M_{st} = Y_{st}L_{st}$ causes helicopter nose-up pitch. The main rotor angle of attack becomes positive, and the air flow approaches the rotor from below. The angle $\Delta\alpha$ of each blade element increases, and the rpm increases, i.e., the main rotor transitions into the autorotative regime.

So far, we have discussed the factors which accelerate or decelerate autorotation of the main rotor. We have devoted considerable attention to this factor, since main rotor rpm in autorotation is the primary index of flight safety. If the rpm is less than the minimal permissible value during autorotation, the rotor can come to a stop — which is a problem which cannot be rectified.

However, during transition into the autorotative regime, the pilot must devote some attention to factors other than main rotor rpm. The helicopter behavior at this time differs markedly from the behavior in steady-state flight:

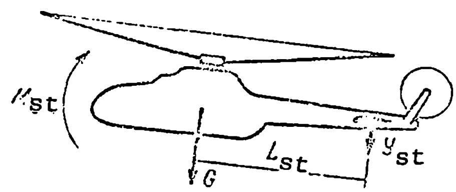

Figure 85. Stabilizer nose-up moment.

First of all, there is a marked reduction of the main rotor reactive moment. As a result of this, the helicopter tends to turn to the right about the vertical axis. Moreover, if there is a horizontal velocity, there will be flapping motions of the blades, and this means that the main rotor coning axis will tilt to the right. As a result of the main rotor thrust force side component, the helicopter will bank and slip to the right.

At the moment of transition into the autorotative regime, the pilot must prevent rotation of the helicopter about the vertical and longitudinal axes by reversing the tail rotor thrust force and deflecting the main rotor cone of rotation to the left. The tilt of the helicopter fuselage relative to the horizon depends on the flight speed. At low speed, the tilt reaches 10-15°, i.e., the nose of the helicopter is quite high. This cannot be permitted, as the helicopter tail rotor may come in contact with the ground, and tail rotor failure may occur. /136

In steady-state autorotation the main rotor blades develop a driving torque. Under the influence of this torque, the blades are rotated forward relative to the vertical hinge to a negative lag angle. During glide, the driving torque depends on the azimuth angle. Therefore, the lag angle will vary, i.e., during glide, the blades will oscillate about the vertical hinges.

§ 65. Gliding Characteristics of Dual-Rotor Helicopters

Gliding of dual-rotor helicopters in the autorotative regime has certain peculiarities in comparison with the single-rotor machines. In the dual-rotor helicopter with tandem rotors, the air flow approaches the forward rotor at a large angle of attack and the aft rotor at a smaller angle (Figure 86a). The

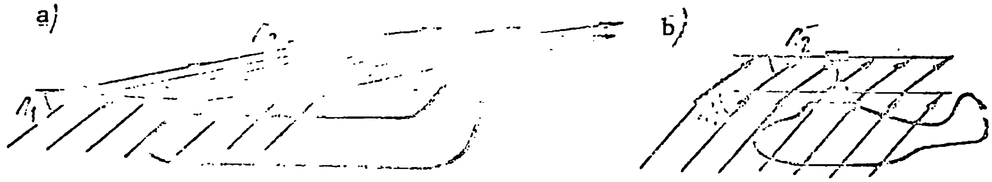

Figure 86. Dual-rotor helicopter gliding characteristics.

angle of attack change takes place because of deflection of the flow by the forward rotor. In the dual-rotor helicopter with coaxial rotors, the lower rotor deflects the flow approaching the upper rotor, which leads to reduction of the angle of attack of the upper rotor (Figure 86b).

The reduction of the angle of attack of the aft rotor in the tandem arrangement and of the upper rotor in the coaxial helicopter leads to reduction of the axial component of the approaching flow and to reduction of $\Delta\alpha$ of the blade elements. Consequently, the aft rotor will operate under conditions of decelerated autorotation. Moreover, along with the angle of attack decrease there is a decrease of the thrust force of the aft rotor. It is necessary that the thrust forces of the two rotors be the same in order to maintain equilibrium of the helicopter. In order to increase the thrust force of the aft rotor, its pitch must be increased, which leads to still greater deceleration of its rotation.

Since the two rotors must rotate in exact synchronism, their autorotative conditions will be different. The front rotor will operate under accelerated autorotative conditions while the aft operates under decelerated conditions, i.e., the front rotor "leads" the aft rotor (it creates the driving torque for the aft rotor). As a result of the driving torque and friction in the trans- /137
mission components, the front rotor develops a yawing moment which causes the helicopter to turn in the direction of rotation of the front rotor. The aft rotor develops a reactive torque, which also causes the helicopter to turn in

the direction of rotation of the front rotor. The helicopter will tend to yaw when gliding. This yawing is eliminated by deflecting the axes of the main rotor cones in opposite directions. Therefore, control of the helicopter during gliding is difficult.

The upper rotor of the coaxial helicopter also operates under decelerated autorotation conditions. Consequently, the lower rotor must develop the driving torque for the upper rotor. Here again, yawing of the helicopter in the direction of rotation of the lower rotor develops. This is eliminated by deflecting the directional control. Control of the coaxial helicopter in a glide is also more complex than control of the single-rotor helicopter.

In conclusion, we note that on all helicopters gliding is performed at a considerably lower speed than used for horizontal flight. The reduction of the gliding speed is explained by the onset of flow separation. Since the flow approaches the main rotor from below, the angles of attack of all the blade elements will be greater than in horizontal flight. Therefore, the blades reach their stalling angle at the $\psi = 270°$ azimuth at a speed considerably lower than in horizontal flight.

Programmed Testing Questions and Answers

Question 1. What factors determine the minimal vertical velocity in steady vertical descent of the helicopter in the main rotor autorotation regime?

Answer 1. In steady-state vertical descent the velocity will be constant. Constancy of the velocity is determined by the first condition for vertical descent $G = T + X$. This equality implies that: the higher the drag of the nonlifting parts of the helicopter, the lower the vertical rate of descent. The thrust force depends in turn on the rpm: the higher the main rotor rpm, the lower the vertical rate of descent.

Answer 2. The vertical velocity is constant in the case of steady-state descent of the helicopter. It is found from the formula

$$V_{des} = \sqrt{\frac{2G}{C_R F \rho}}.$$

Therefore, the vertical velocity depends on helicopter weight, main rotor pitch, area swept by the rotor, and air density.

Answer 3. The steady-state vertical rate of descent in the autorotative regime along a vertical trajectory is found from the approximate formula

$$V_{des} = 3.6\sqrt{P}.$$

Therefore, the vertical velocity depends on the disk loading ($P = G/F$) or the helicopter weight. The higher the disk loading, the higher the vertical velocity. At high latitude the vertical velocity will be greater than at sea level and increases with increase of the altitude. /138

Question 2. What determines the blade element autorotation conditions?

Answer 1. The blade element autorotation conditions are determined by the inclination of the elemental aerodynamic force relative to the main rotor hub axis. If the force is directed parallel to the hub axis, its projection on the plane of rotation equals zero, and the autorotation will be steady-state. If the force vector ΔR is inclined forward relative to the hub axis by the angle γ, the autorotation will be accelerated. If the force vector ΔR is inclined aft by the angle γ, the autorotation will be decelerated. The inclination of the vector ΔR depends on the blade element pitch and angle of attack increment($\Delta\alpha$ = arc tg V_{des}/u). The lower the pitch and the larger $\Delta\alpha$, the larger the forward tilt of the vector ΔR.

Answer 2. The blade element autorotation conditions are determined by the aerodynamic efficiency $K = Y/X$. The higher the aerodynamic efficiency, the smaller the aerodynamic efficiency angle θ_k, the larger the forward tilt of the force vector ΔR, and the higher the main rotor rpm will be. Since the

aerodynamic efficiency and the efficiency angle depend on the angle of attack, the autorotation conditions are determined by the angle of attack. At the optimal angle of attack, the force vector ΔR has the maximal forward tilt, therefore, the autorotative regime will be accelerated. At angles of attack greater than and less than the optimal value, the blade element autorotation will be decelerated.

Answer 3. The blade element autorotative conditions are determined by the tilt of the elemental resultant aerodynamic force R relative to the main rotor hub axis. If this force vector is tilted forward, the blade element will have accelerated autorotation; if it is tilted aft the autorotation will be decelerated. If the direction of ΔR is parallel to the hub axis, the autorotation is steady-state. The tilt of the force ΔR depends on the angle of attack increment $\Delta\alpha$ formed as a result of the vertical rate of descent ($\Delta\alpha = \text{arc tg } V_{des}/\omega r$). The higher the vertical rate of descent, the larger $\Delta\alpha$, the larger the tilt of the force ΔR, and the higher the main rotor rpm.

Question 3. Variation of autorotation conditions for the different blade elements.

Answer 1. The different blade elements have different autorotation conditions. These conditions are determined by the geometric twist of the blade, i.e., by the magnitude of the blade element incidence angle and the magnitude of the angle of attack increment caused by the vertical rate of descent. The incidence angles are larger for the root elements than for the tip elements. Increase of the incidence angles leads to deceleration of the autorotation.

The angle of attack increment ($\Delta\alpha = \text{arc tg } V_{des}/\omega r$) depends only on r, which means that its magnitude is larger for the root elements. The larger $\Delta\alpha$, the more accelerated the autorotation will be. The effect of the angle of attack increment on autorotation is greater than the influence of geometrical twist, therefore, the root elements will have accelerated autorotation while the tip elements will have decelerated autorotation.

Answer 2. The autorotation conditions are different for the different blade elements. The autorotation conditions are determined by the tilt of the force vector ΔR, and this tilt, in turn, depends on the pitch of the given element. Consequently, the autorotation conditions are determined by the pitch of the given element. As a result of the geometric twist, each element has its own pitch. The pitch for the root elements is larger, and this means that for these elements the force vector ΔR is tilted aft more, and the autorotation is decelerated. The tip elements have lower pitch, therefore, they /139
have accelerated autorotation.

Answer 3. The autorotation conditions of the different blade elements are determined by the geometric twist, circumferential velocity ωr of the blade element, and the induced velocity. As a result of geometric twist, the root blade elements have more pitch and higher induced velocity, therefore, they will have a lower vertical flow velocity. As a result of the higher incidence angle and lower vertical velocity, there is a reduction of the angle of attack increment ($\Delta\alpha$ = acr tg $V_{des.e}/\omega r$), and, therefore, the force vector ΔR is tilted aft. The conclusion is that the autorotation of the root elements will be decelerated, while that of the tip elements is accelerated.

Question 4. Azimuthal variation of the blade element autorotation conditions in helicopter gliding.

Answer 1. During helicopter gliding in the main rotor autorotative regime, the flow conditions about the blade element change continuously. Therefore, the autorotation conditions will also change. The resultant velocity ($W = u + V \sin\psi$) will increase continuously for the advancing blade. This leads to increase of the elementary force ΔR and to acceleration of the autorotation. The resultant velocity of each blade element of the retreating blade decreases and reaches the minimum ($W = u - V$) at the $\psi = 270°$ azimuth. Therefore, the force ΔR also decreases, and the autorotation will be decelerated. Each blade element becomes alternately "driving", then "driven". Most of the elements of the advancing blade will be "driving"; most of those of the retreating blade will be "driven".

Answer 2. During helicopter gliding, the autorotation conditions of each element depend on the blade azimuth. With change of the azimuth there is a change of the element resultant velocity ($W=u+V \sin \psi$). At the 90° azimuth this velocity reaches its maximal value, therefore, the angle of attack increment is minimal ($\Delta\alpha = \text{arc tg } V_{gl} \sin \theta/\omega r + V_{gl} \cos \theta$). The force vector ΔR tilts aft, and the autorotation will be decelerated.

At the 270° azimuth the element resultant velocity is minimal ($\Delta\alpha = \text{arc tg } V_{gl} \sin \theta/u - V_{gl} \cos \theta$). The forward tilt of the force vector ΔR will be maximal, and the autorotation will be accelerated. The conclusion is that during gliding the retreating blade creates a driving torque and "drives" the advancing blade, which develops a retarding torque.

Answer 3. The blade element characteristics during helicopter gliding are determined by two factors: azimuthal variation of the resultant flow velocity over the blade element, and the presence of flapping motions and vertical flapping velocity.

At the 90° azimuth the resultant velocity is maximal; the vertical flapping velocity is also maximal and directed upward. The angle

$$\Delta\alpha = \text{arctg} \frac{V_{gl} \sin \theta - V_{fl}}{u + V_{gl} \cos \theta}$$

is minimal; therefore, the force vector ΔR is tilted aft, and the autorotation will be decelerated.

At the 270° azimuth the resultant flow velocity is minimal, and the vertical flapping velocity is maximal, but directed downward. The angle

$$\Delta\alpha = \text{arctg} \frac{V_{gl} \sin \theta + V_{fl}}{u - V_{gl} \cos \theta}$$

is maximal; therefore, the force vector ΔR is directed forward, and the blade element autorotation will be accelerated.

The conclusion is that the retreating blade develops a driving torque while the advancing blade develops a retarding torque, but the rotor autorotation will be steady-state.

Question 5. What determines the minimal vertical rate of descent during helicopter gliding? /140

Answer 1. The vertical rate of descent descent during helicopter gliding is found from the formula $V_{des} = V_{gl} \sin \theta$, i.e., it depends on the velocity along the trajectory and the gliding angle. The velocity along the trajectory depends on the forward or aft tilt of the coning axis. The gliding angle is determined by the lift force and depends on the main rotor pitch: the larger the pitch, the larger the lift force, and the smaller the gliding angle and the vertical rate of descent.

Answer 2. During helicopter gliding the vertical rate of descent is found from the formula $V_{des} = V_{gl} \sin \theta$. Since during a glide the propulsive force is the weight force $G_2 = G \sin \theta$, the work of the weight force per unit time will be the equivalent power supplied to the rotor

$$G_2 V_{gl} = N_h = G \sin \theta \, V_{gl} = G V_{des}.$$

Hence

$$V_{des} = \frac{N_h}{G}; \; V_{des_{min}} = \frac{N_{h_{min}}}{G}.$$

Thus, the minimal vertical rate of descent in a glide depends on the flight weight and altitude. The gliding velocity along the trajectory should be equal to the economical speed for horizontal flight.

CHAPTER X

HELICOPTER TAKEOFF AND LANDING

§ 66. Takeoff

Helicopter takeoff is an unsteady accelerated flight mode. During takeoff the velocity varies from $V = 0$ to the velocity at which steady-state climb is established. This climbing speed is usually equal to the economical horizontal flight speed. Depending on takeoff weight, airfield altitude above sea level, and presence of obstacles, the takeoff may be performed helicopter-style, airplane-style, and helicopter-style with or without utilization of the "air cushion."

Sometimes the helicopter travels over the ground prior to or during takeoff, i.e., taxiing is performed. Helicopter taxiing differs significantly from airplane taxiing.

Helicopter taxiing characteristics. Taxiing is accomplished by means of the propulsive force P, which balances the wheel friction force F_{fr} (Figure 87a). The reactive moment of the main rotor is balanced by the thrust moment of the tail rotor. The basic differences in helicopter taxiing are:

(1) Presence of a large lift force, which is a component of the main rotor thrust and reduces the wheel pressure force on the ground, i.e., reduces the support reaction. As a result, wheel friction on the ground is reduced, and the possibility of helicopter overturning is increased; /141

(2) Presence of side forces: tail rotor thrust and the side component of the main rotor thrust (Figure 87b). These forces develop large overturning moments about the wheel support points, which balance one another. But if there is a change of one of the side forces the overturning moment is unbalanced and can cause the helicopter to overturn (Figure 87c);

(3) A large nose-down moment develops as a result of the propulsive force P, which creates high loads on the landing gear wheels (wheel).

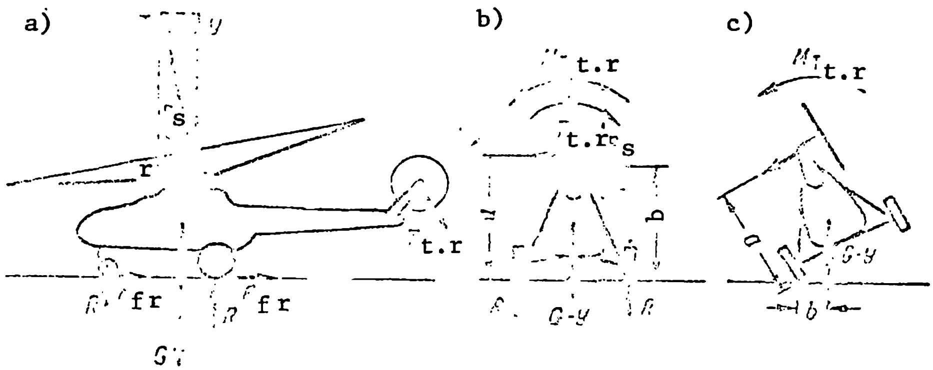

Figure 87. Forces acting on helicopter in taxiing.

Therefore, helicopter taxiing must be performed more carefully than airplane taxiing. The taxiing speed must not exceed 10 - 15 km/hr. The surface of the area over which taxiing is performed must be smooth. Taxiing in a strong crosswind is not permitted, since this can lead to overturning of the helicopter.

Helicopter-style takeoff is the primary takeoff mode (Figure 88). In this takeoff a vertical liftoff is made and check hovering is performed at a height of 1.5 - 2 m (operation of the main rotor, engine, and equipment is checked). Then the helicopter is transitioned into climb along an inclined trajectory with simultaneous increase of the speed. In this process "sinking"

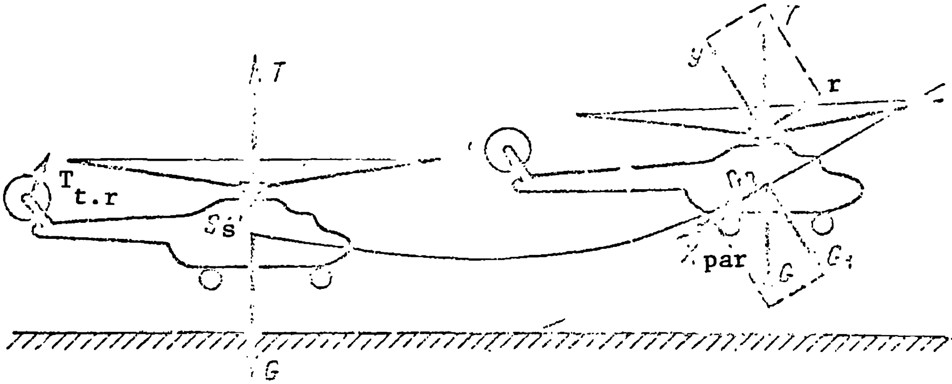

Figure 88. Helicopter-type takeoff.

of the helicopter is possible, i.e., a reduction of the altitude, and sometimes the wheels may even come in contact with the ground. This phenomenon is caused by tilting the main rotor coning axis forward to develop the propulsive force P, the result being a decrease of the vertical component of the main rotor thrust. Therefore, along with tilting of the main rotor coning axis forward, there must be an increase of the thrust force by increasing the rotor pitch.

The takeoff is considered terminated when the helicopter reaches a height /142
of 20 - 25 meters or is above the surrounding obstacles. At this time the acceleration, i.e., the increase of the velocity along the trajectory to the optimal climbing speed, which corresponds to the minimum level flight power, is also terminated. But this type of takeoff cannot be performed if:

the helicopter is overloaded (insufficient engine for hovering outside the "air cushion" influence zone);

the air temperature is high (reduced engine power);

the takeoff is made from a high-altitude airfield (low air density at the given altitude so that insufficient engine power is available). Under

these conditions an airplane-type takeoff is made.

<u>Airplane-type takeoff.</u> During the airplane-type takeoff the helicopter accelerates on the ground, then lifts off and transitions into a climb along an inclined trajectory (Figure 89). In this takeoff use is made of the primary advantage of main rotor operation in the forward flight: increase of the thrust developed by the rotor with increase of the velocity of the air stream approaching the main rotor (see Figure 68).

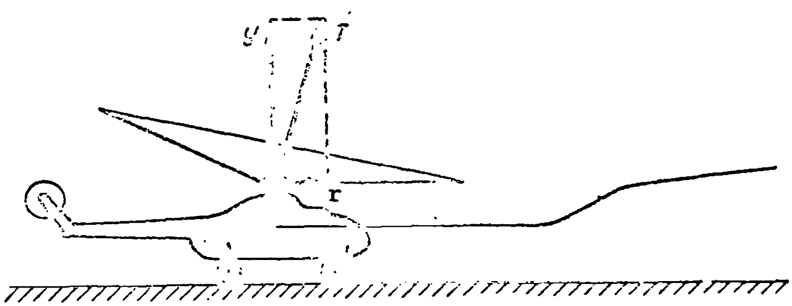

Figure 89. Airplane-type takeoff.

As a result of the thrust increase there is an increase of the lift force. /143 When it becomes somewhat greater than the weight force, the helicopter lifts from the ground and transitions into a climb along an inclined trajectory with further increase of the flight speed. We see from the power required and available curves for horizontal flight (see Figure 63a) that the power required for horizontal flight decreases markedly for even a small speed increase. If takeoff is impossible at V = 0 because of insufficient power, at a speed of 40 - 50 km/hr considerable excess power is developed, which then makes it possible for the helicopter to transition to the climb regime with simultaneous increase of the flight speed.

An airfield or at least a small smooth area is required for the airplane-type takeoff. The ground run during takeoff with flight weight exceeding by

10 - 15% the normal takeoff weight for helicopter-type takeoff is 50 - 100 meters. In this case the liftoff speed is 50 - 70 km/hr (with acceleration during the ground run 2.2 m/sec^2) and the ground run time is 7 - 10 seconds.

The ground run is performed on all wheels of the landing gear. Some helicopters (the Mi-6, for example) perform the last part of the ground run on the nosewheel. When using this ground run technique the acceleration is increased as a result of the inclination of the fuselage longitudinal axis and the resulting increase of the propulsive force P. The helicopter takeoff is considered complete when a safe height (25 m) and a velocity along the trajectory close to the economical speed for horizontal flight have been reached.

Helicopter-type takeoff utilizing the air cushion. Vibrations may arise during airplane-type takeoff ground run on an uneven surface. Then the takeoff is made using the air cushion (Figure 90). In this takeoff the helicopter lifts off vertically, utilizing the increased main rotor thrust in the air cushion influence zone (the distance from the main rotor plane of revolution to the ground does not exceed R).

After liftoff and hovering in the air cushion zone, the helicopter is transitioned into forward flight i.e., flight at low height with increase of the speed. During the transition maneuver the influence of the air cushion diminishes with increase of the speed, but the forward flight effectiveness increases; therefore, the main rotor thrust force increases, which makes it /144 possible to transition the helicopter into a climb along an inclined trajectory. In order to perform such a takeoff it is necessary to have a sufficiently smooth area, i.e., there must not be any large ditches or dropoffs, where the influence of the air cushion disappears.

In certain cases none of the techniques examined above are applicable because of obstacles surrounding the area. Then takeoff is made without

utilization of the air cushion, i.e., liftoff and check hovering are performed and then a vertical climb is initiated. At a height of 5 - 10 meters above the surrounding obstacles the helicopter is transitioned into climb along an inclined trajectory with simultaneous acceleration to the economical velocity. Vertical takeoff is rarely used, since it requires high power and is performed in the danger zone. If sufficient power is not available, yet takeoff must be made, the helicopter weight should be reduced.

§ 67. Landing

Landing is transitional flight from a height of 25 - 50 m with reduction of the velocity and subsequent touchdown. Helicopter landing may be performed in helicopter style, in airplane style, in the autorotative regime, from a glide along an inclined trajectory, and with flare-out.

The helicopter-style landing is the primary technique for landing with the engine operating. It includes the following stages (Figure 91):

(1) glide with reduction of the speed along the trajectory and vertical rate of descent;

(2) hover at a height 2 - 3 m above the landing area;

(3) vertical descent;

(4) touchdown.

During the landing approach the helicopter performs steady-state descent along an inclined trajectory with the engine operating. At a height of 40 - 50 m reduction of the speed along the trajectory is initiated while maintain- /145
ing a constant descent angle. In this stage the motion of the helicopter is governed by the following conditions:

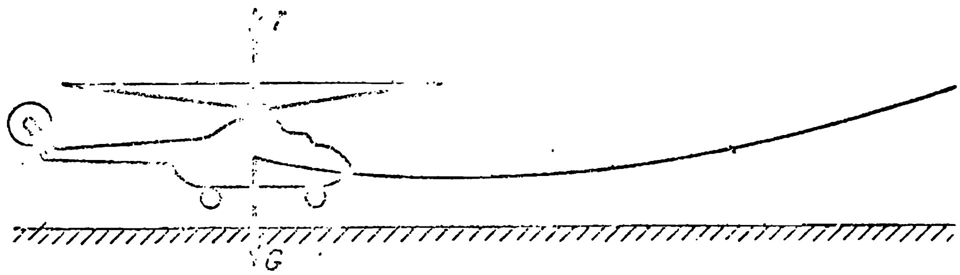

Figure 90. Takeoff on air cushion.

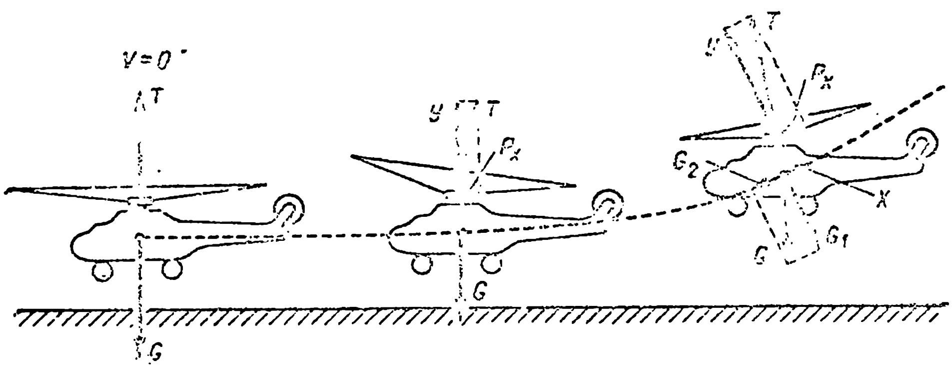

Figure 91. Helicopter-type landing.

$Y = G_1 = G \cos \theta$ (constant descent angle);

$P_x + X_{par} > G_2$ (reduction of the speed);

$T_{t.r} = S_s$ (absence of lateral displacement);

$M_r = M_{t.r}$ and $\Sigma M_{cg} = 0$ (constant direction of flight, i.e., absence of rotation about the helicopter's primary axes).

Deceleration of the helicopter is achieved by tilting the main rotor thrust force vector aft and increasing the thrust component P_x. Upon reaching

a speed of 50 - 60 km/hr, the vertical rate of descent is reduced by increasing main rotor pitch and its thrust force. The helicopter deviates from the descent path angle and travels parallel to the surface of the ground at a height of 2 - 3 m. During this inertial motion the speed decreases to zero and the helicopter hovers above the landing area, orienting itself relative to the center of the area. If the landing approach was not made directly into the wind, the helicopter is turned about the vertical axis to take up a heading into the wind. Then a vertical descent is made at a low rate in order to avoid rough contact of the wheels with the ground.

The airplane-type landing is made under the same conditions as the take-off of the same type. It includes the following stages (Figure 92): glide from a height of 15 - 30 m, flare, holdoff, touchdown, and rollout. During the descent altitude is lost, but a constant speed and descent angle are maintained. The descent is made with the engine operating. The flare is initiated at a height of 7 - 10 m by increasing the thrust and lift forces.

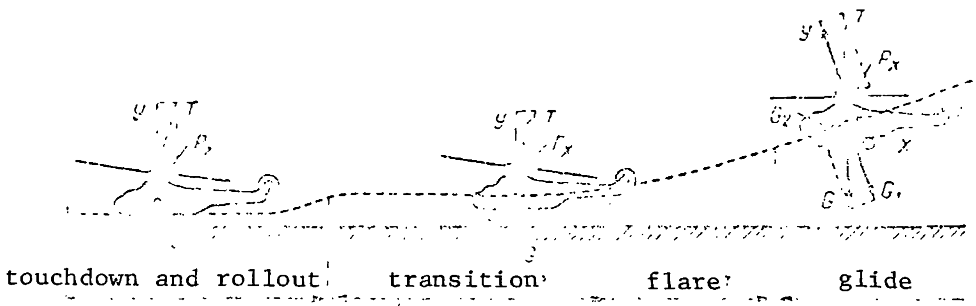

Figure 92. Airplane-type landing.

The flare is terminated at a height of 1 - 0.5 m and the horizontal component of the velocity diminishes at this point, since the weight force component G_2 decreases to zero. After the flare the helicopter still has a relatively high speed, which is then reduced during the holdoff period. The

touchdown is made on the main gear wheels at a speed of 30 - 40 km/hr. In this procedure care must be taken that the tail does not get too low, since damage to the tail skid and the tail rotor can occur. The touchdown is followed by rollout, during which the main rotor thrust is decreased. An airfield or a smooth area with firm soil is necessary for the airplane-type landing. /146

Landing under special conditions. If the landing area is surrounded by obstacles, a helicopter-type landing is made without utilization of the "air cushion." Hovering is performed into the wind at a height of 5 - 10 m above the obstacles to align the helicopter with the center of the area. Then a vertical descent is made at a rate of descent of no more than 2 m/sec in order to avoid the vortex ring state. During this descent the vertical velocity of the helicopter is reduced to 0.2 - 0.3 m/sec at the moment of touchdown. Therefore, this type of landing can only be made if there is sufficient power margin available for hovering outside the "air cushion" influence zone. This type of landing is used only in case of extreme urgency, since a safe landing is not guaranteed in case of engine failure at a height of more than 10 m (in the danger zone).

Landing in the main rotor autorotation regime with glide along an inclined trajectory. We have established above that in case of engine failure flight in the main rotor autorotation regime should be made along an inclined trajectory rather than vertically.

Landing from a glide along an inclined trajectory is similar to the landing of an airplane and requires a smooth area with solid ground. It consists of the following stages (Figure 93):

(1) glide at constant angle and constant speed;

(2) deceleration (reduction of glide angle and vertical rate of descent through use of the kinetic energy of the helicopter and the main rotor);

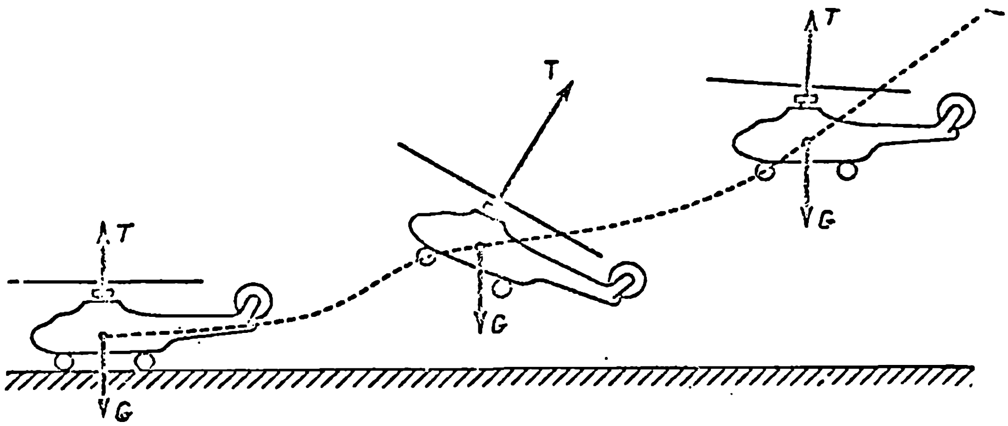

Figure 93. Autorotative landing.

(3) touchdown;

(4) rollout and reduction of main rotor pitch.

The advantage of this landing is that the helicopter has considerably less /147 vertical velocity and better controllability. In addition, during flight along the inclined trajectory the helicopter has considerable kinetic energy, which is used to reduce the vertical velocity prior to touchdown. Therefore, the landing is safer and simpler when the helicopter approaches in a glide along an inclined path. This type of landing utilizes the kinetic energy of the entire helicopter, as well as the kinetic energy of the main rotor.

During the glide a constant descent angle is assured by the condition

$$Y = G_1 = G \cos \theta,$$

and constant velocity is assured by the condition (see Figure 92)

$$G_2 = X_{par} + P_x.$$

The gliding speed is close to the economical speed for level flight, but it changes as a function of the wind velocity and direction. When gliding into the wind the helicopter speed should be higher, the higher the wind velocity.

The height for initiation of the deceleration or flare is different on different helicopters: the higher the disk loading, the higher this height. For example, this height for the Mi-1 is 15 - 20 m; for the Mi-4 it is 25 - 20 m. Deceleration is obtained by tilting the main rotor coning axis aft. This leads to increase of the main rotor angle of attack and increase of the angle of attack of each blade element, which then leads to increase of the main rotor thrust force and its rpm. Therefore, both the glide angle and the velocity along the trajectory decrease. After the coning axis is deflected aft, the main rotor collective pitch must be increased to the maximal value. This leads to further increase of the thrust force and reduction of the vertical rate of descent. The helicopter will travel for some time parallel to the ground surface, similar to the motion of an airplane during transition. The height at the end of this motion decreases to 0.5 - 0.3 m and the helicopter touches down with vertical velocity close to zero.

During the flare and transition the helicopter nose is high. Touchdown cannot be made in this attitude because of danger of damage to the tail rotor. Therefore, prior to touchdown the helicopter's nose is lowered by deflecting the cyclic control stick forward. The increase of main rotor pitch leads not only to increase of the thrust force, but also to increase of the resistance to rotation; the rotor rpm decreases and the coning angle increases (blades flap up). If the rotor pitch is not reduced after touchdown, the rotor blade may descend abruptly and strike the tail boom.

Landing in the autorotative regime with vertical descent. If the landing /148
is made on a small area which is surrounded by obstacles, the landing must be made from a vertical descent. It was established earlier that the vertical rate of descent during flight in the autorotative regime along a vertical

trajectory is found from the formula $V_{des} = 3.6\sqrt{P}$ and amounts to 14 - 20 m/sec. Touchdown at such a velocity leads to damage to the helicopter and does not guarantee safety of the crew. Therefore, the velocity is reduced prior to touchdown by utilizing the kinetic energy of the main rotor. Such a landing is called a landing with flare-out. The essence of the operation is as follows.

At a height of 20 - 25 m the main rotor pitch is increased to the maximal value, and the main rotor rpm should be as large as possible in order to impart maximal kinetic energy to the main rotor (the kinetic energy is proportional to ω^2). As the pitch is increased there is a marked increase of main rotor thrust flare-out, which leads to reduction of the vertical rate of descent to 3 - 5 m/sec. Such a velocity can be abosrbed completely by the gear shock absorbers,and a safe landing can be made.

The landing is easier when there is a wind. In this case the helicopter is turned into the wind and transitioned into the inclined glide regime (slope 45°). The effect of the wind is to carry the helicopter aft, and its trajectory relative to the ground will be nearly vertical. The helicopter controllability is better in this type of glide, and the thrust force is increased somewhat as a result of forward flight.

A vertical landing in the autorotation regime requires considerable skill and coolness on the part of the pilot. The following errors are possible in this type of landing:

(1) early flare-out (reduction of vertical velocity at a high altitude);

(2) late flare-out, as a result of which the vertical velocity is not reduced and hard contact with the ground may result.

It is clear from this discussion that the velocity is not entirely arrested in landing from a vertical descent. Even when the flare-out is

performed correctly, the final velocity may be quite large — 3 - 5 m/sec or more. This is explained by the fact that the helicopter, which has a rate of descent prior to flare-out of 15 - 20 m/sec, has considerable kinetic energy $\left(E_k = \frac{GV_{des}^2}{2g}\right)$. An equivalent kinetic energy of the main rotor must be expended to arrest the vertical velocity completely. But the main rotor kinetic energy is not entirely utilized for braking the helicopter. A large part of the rotor energy is expended in overcoming the profile and induced drags, on friction in the transmission, and on the blade end losses. Only one fifth or sixth of the total kinetic energy of the main rotor is used for deceleration. This means that the main rotor must have 5 - 7 times the kinetic /149 energy of the helicopter in order to decelerate the helicopter completely. In reality the main rotor kinetic energy is about three times the helicopter kinetic energy.

Therefore, the vertical velocity cannot be arrested completely, but it must be reduced as much as possible, and to this end the main rotor rotational energy is increased; this energy is proportional to the blade mass and the square of the angular velocity of revolution

$$E_\omega = \frac{j\omega^2}{2},$$

where E_ω is the rotational kinetic energy;

j is the main rotor moment of inertia;

$$j = \frac{G_b k}{3g} R^2.$$

Substituting the value of the rotational moment of inertia into the formula, we obtain

$$E_\omega = \frac{G_b k R^2}{6g} \omega^2 = \frac{G_b k u^2}{6g}.$$

We see from the formula that heavier blades are required for safe landing in the autorotation regime.

CHAPTER XI

HELICOPTER BALANCE, STABILITY, AND CONTROL

§ 68. Helicopter Center of Gravity and Balance

The helicopter center of gravity is the point of application of its weight force vector. The center of gravity is the nominal point about which the helicopter rotates. The three principal axes of rotation (body coordinate system) passing through the helicopter center of gravity are used to characterize the rotational motions (Figure 94a). The $0 - x_1$ longitudinal axis lies in the plane of symmetry and runs along the fuselage parallel to the main rotor hub rotation plane. The $0 - z_1$ transverse axis passes through the center of gravity perpendicular to the plane of symmetry and is directed to the right. The $0 - y_1$ vertical axis passes through the center of gravity, lies in the plane of symmetry perpendicular to the longitudinal axis, and is directed upward. /149

If the external force acting on the helicopter passes through the helicopter center of gravity, its moment will be zero and the helicopter will not have any rotational motion. If the external force passes outside the center of gravity, it creates a moment relative to some axis, under the influence of which the helicopter will rotate. /150

Of the cargo is attached rigidly to the helicopter the center of gravity does not move regardless of the attitude the helicopter assumes in the air. If the cargo moves, the center of gravity will also move. Therefore, we need

to know precisely where the helicopter center of gravity is located. The center of gravity location is determined by balancing the helicopter. The helicopter balance point is the distance x from the main rotor hub axis to the center of gravity, expressed in millimeters, and the distance y from the center of gravity to the hub rotation plane (Figure 94b).

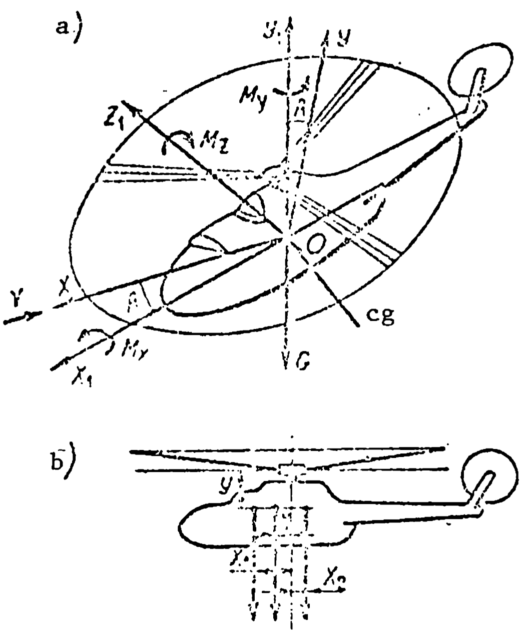

Figure 94. Helicopter balance:
a - helicopter cg definition;
b - helicopter balance: x_1 - limiting forward; x_2 - limiting aft.

The distance x is the horizontal cg location and the distance y is the vertical cg location.

If the center of gravity is located ahead of the hub axis the cg is termed forward and denoted by +x. If the center of gravity is located behind the hub axis it is termed aft and denoted by -x. Every helicopter has strictly defined cg travel limits. The forward cg limit is considerably greater than the aft limit.

For example, for the Mi-1 the forward cg limit is $+x_{lim}$ = 150 mm, the aft limit is $-x_{lim}$ = -53 mm.

The helicopter cg location must be known prior to every flight. The cg location changes with variation of helicopter loading. The locations where the heaviest cargo is to be located is indicated in the operating manual for every helicopter. This manual also defines the sequence for finding the cg location, which amounts to the following. The basic helicopter weight (weight at a definite loading) and the basic cg location must be known. These data are presented in the helicopter specifications. Moreover, the weights and locations of the cargo must be /151

known. The distance from the main rotor hub axis to the cargo is measured in meters. The total moment about the main rotor hub axis is calculated and the new helicopter weight is determined as the sum of the basic weight and the weights of all the cargo. The new cg location is found from the formula

$$x = \frac{\Sigma M}{\Sigma G}.$$

Example of cg calculation of Mi-1 using the data:

(1) basic helicopter weight 1930 kgf;

(2) basic cg location 123 mm;

(3) cg limits +150, -53;

(4) additional cargo on helicopter:

G_1 = 85 kgf (at distance l_1 - 1.2 m ahead of hub axis);

G_2 = 38 kgf (at distance l_2 = 1.4 m behind hub axis);

G_3 = 105 kgf (at distance l_3 = 0.5 m ahead of hub axis);

(5) cargo G_4 = 72 kgf removed, was located at distance l_4 = 0.6 m aft of hub axis.

Solution. We find the moments of the basic weight and the weight of each cargo

$$M_{bas} = G_{bas} X_{bas} = 1930 \cdot 0.123 = 238 \text{ kgf} \cdot \text{m};$$
$$M_1 = 85 \cdot 1.2 = 104 \text{ kgf} \cdot \text{m};$$
$$M_2 = 38 \cdot (-1.4) = -53 \text{ kgf} \cdot \text{m};$$
$$M_3 = 105 \cdot 0.5 = 52 \text{ kgf} \cdot \text{m};$$
$$M_4 = (-72) \cdot (-0.6) = 43 \text{ kgf} \cdot \text{m};$$
$$\Sigma M = 238 + 104 - 53 + 52 + 43 = 384 \text{ kgf} \cdot \text{m};$$
$$\Sigma G = 1930 + 85 + 38 + 105 - 72 = 2086 \text{ kgf}.$$

We find the new cg location

$$x_{new} = \frac{\Sigma M}{\Sigma G} = \frac{384}{2086} = 0{,}184 \ \text{m}.$$

Thus the cg location of the helicopter is outside the forward cg limit by the distance

$$\Delta x = 184 - 150 = 34 \ \text{mm}.$$

Consequently, flight cannot be made with this cg location; the helicopter will be uncontrollable. Some of the cargo must be shifted aft.

Let us find how far the cargo G_{car} = 105 kgf must be moved aft to locate the cg at +150 mm.

Solution.

1. We find the moment required to shift the cg 34 mm

$$\Delta M = G_{new}\Delta x = 2086 \cdot 0.034 = 70.9 \ \text{kgf}.$$

2. We find the distance which the cargo must be shifted

$$l = \frac{\Delta M}{G_{car}} = \frac{70.9}{105} = 0.67 \ \text{m}.$$

§ 69. General Analysis of Helicopter Equilibrium /152

That state of the helicopter for which it travels in a straight line with constant velocity and does not rotate about its principal axes (about the center of gravity) is called equilibrium.

The equilibrium conditions follow from the definition. According to Newton's first law, a body moves uniformly and rectilinearly if no external forces act on it. Therefore, it is necessary that the sum of the forces acting on the helicopter be equal to zero

$$\Sigma F_{cg} = 0.$$

The second equilibrium characteristic (absence of rotation) will hold if the sum of the moments of the forces acting on the helicopter equals zero

$$\Sigma M_{cg} = 0.$$

Moments relative to the o - z transverse axis are termed longitudinal (M_z). Under the action of this moment the helicopter pitches up (nose rises) or pitches down (nose descends). The moments about the $0 - x_1$ longitudinal axis are termed transverse or rolling moments (M_x). The moments about the $0 - y_1$ vertical axis are termed directional (M_y). A general remark on the sign of the moments: a positive moment causes clockwise helicopter rotation if we look along the direction of the axis.

Equilibrium of the helicopter exists in all the steady-state flight regimes. The steady-state flight conditions, which we examined previously, are the equilibrium conditions written in expanded form. It is true that these conditions were written in application to the velocity coordinate system. The velocity or wind coordinate system is a system fixed with the flight velocity vector. In this system the longitudinal axis is denoted by 0 - x and coincides in direction with the velocity vector (see Figure 94a). The angle between the axes $0 - x_1$ and 0 - x of the body and wind coordinate systems is equal to the main rotor angle of attack A. The angle between the longitudinal axis of the velocity coordinate system and the helicopter plane of symmetry is called the sideslip angle. If the flight velocity vector is in the plane of symmetry,the sideslip angle equals zero. In the absence of sideslip,the transverse axes of the body and velocity coordinate systems

coincide. The angle between the vertical axes $0 - y_1$ and $0 - y$ of the body and velocity coordinate system equals the angle of attack of the main rotor.

We take for example the conditions for horizontal helicopter flight /153

$$Y = G \text{ or } Y - G = 0;$$
$$P = X_{par} \text{ or } P - Z_{par} = 0;$$
$$T_{t,r} = S_s \text{ or } T_{t,r} - S_s = 0.$$

We see from these equalities that the sum of the forces acting on the helicopter along the vertical, longitudinal, and transverse axes of the velocity coordinate system equals zero.

Consequently, these three equalities express the first equilibrium condition $\Sigma F_{cg} = 0$. The fourth horizontal flight condition ($\Sigma M_{cg} = 0$) expresses the second equilibrium characteristic, i.e., the absence of rotation about the center of gravity.

§ 70. Helicopter Equilibrium in the Hovering Regime

The conditions of helicopter equilibrium in the hovering regime can be applied (with some additions) to any flight regime .

Hub horizontal hinge offset. Most modern helicopters have offset of the main rotor hub horizontal hinges, i.e., separation between the hub axis and the horizontal hinge axis, which is denoted by l_{hh} (Figure 95a). Horizontal hinge offset has an effect on helicopter equilibrium, stability, and controllability conditions.

The centrifugal forces acting on the rotor blades are transmitted to the horizontal hinges. When the main rotor plane of rotation is parallel to the hub plane (no tilt of the coning axis), the blade centrifugal forces are in

a single plane and their moment relative to the center of the hub equals zero.

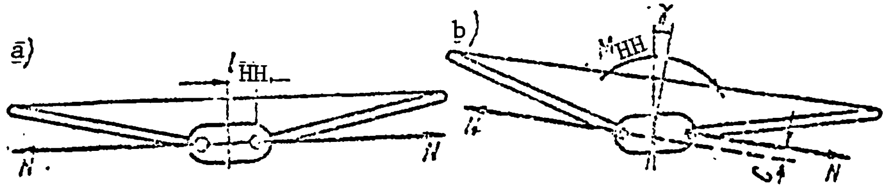

Figure 95. Horizontal hinge offset

If the main rotor coning axis is deflected from the hub axis, the main rotor plane of rotation will not be parallel to the hub rotation plane. The blade centrifugal forces act in a plane parallel to the main rotor plane (Figure 95b).

If the horizontal hinges are offset, there will be an arm c between the centrifugal forces; therefore, these forces create the moment $M_{hh} = Nc$ /154
relative to the center of the hub. This moment rotates the main rotor hub and consequently the entire helicopter so as to make the hub axis approach the coning axis.

Equilibrium of the helicopter relative to the principal axes of rotation can be subdivided into longitudinal, transverse, and directional. Common to all these modes is the first equilibrium characteristic: uniform rectilinear motion or, as a particular case, relative rest in the hovering regime ($V = 0$). Therefore, in the definitions of the equilibrium modes we omit the first characteristic, assuming that it holds.

<u>Helicopter longitudinal</u> equilibrium is that state of the helicopter in which it does not rotate about the transverse axis. Since the velocity in the hovering regime equals zero, there will be no forces parallel to the helicopter longitudinal axis. Then the first equilibrium characteristic is

expressed by the two equalities

$$T \approx G \text{ or } T - G = 0;$$

$$T_{t.r} = S_s \text{ or } T_{t.r} - S_s = 0.$$

The second equilibrium condition is that the sum of the longitudinal moments must equal zero: $\Sigma M_z = 0$.

Longitudinal moments are created by (Figure 96):

main rotor thrust force ($M_T = Ta$);

stabilizer lift force ($M_{st} = Y_{st}L_{st}$);

horizontal hinge moment ($M_{hh} = NC$);

tail rotor reactive moment $M_{r_{t.r}}$.

If the cg location is far forward the helicopter will hover with the nose down (Figure 96a). In this case the main rotor thrust force moment will be /155
nose-down; the moments of the horizontal hinges, stabilizer, and the tail rotor reactive moment are nose-up. Therefore, the second longitudinal equilibrium characteristic is expressed by the equation

$$M_{hh} + M_{cr} + M_{r_{t.r}} = M_t$$

or

$$NC + Y_{st}L_{st} + M_{r_{t.r}} - Ta = 0.$$

To satisfy this condition the main rotor coning axis must be tilted aft. The more forward the cg, the larger the coning axis deflection angle η must be. If the cg moves forward beyond the limiting forward position, hovering longitudinal equilibrium cannot be achieved.

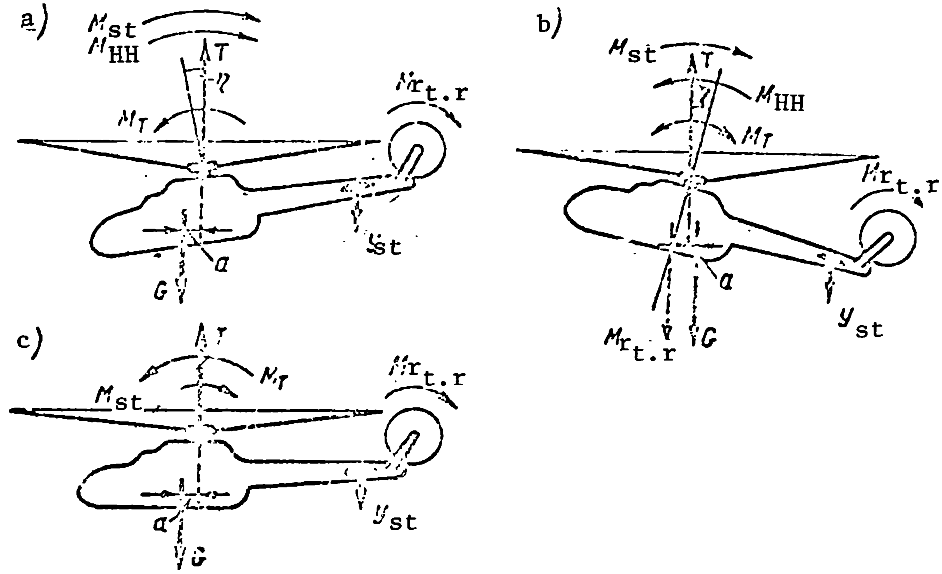

Figure 96. Helicopter longitudinal equilibrium in hover.

If the helicopter cg is aft or slightly forward, the helicopter will hover with the nose high (Figure 96b). In this case the main rotor coning axis must be deflected forward relative to the hub axis through the angle η. The moment of the main rotor thrust force may be positive, negative, or zero. The moment of the stabilizer and tail rotor, as in the first case, will be nose-up. The second equilibrium characteristic will be expressed by the equality

$$M_{st} + M_{r_{t.r}} \pm M_T - M_{hh} = 0.$$

If the helicopter cg moves aft beyond the permissible limit, the main rotor coning axis is deflected full forward. In this case the helicopter cannot be transitioned into the horizontal flight regime, and if this cannot be done the flight speed cannot be increased to the maximal value.

Helicopter hover in the horizontal attitude is possible with a slightly forward cg location. In this case the main rotor coning axis will coincide

with the hub axis. The moment of the horizontal hinges will be zero. The main rotor thrust force will be nose-down (Figure 96c). The longitudinal equilibrium condition is expressed by the equality

$$M_{st} + M_{r_{t.r}} - M_T = 0.$$

Longitudinal equilibrium of the helicopter in other flight regimes as a function of cg location can be expressed by one of the equalities discussed above. However, to the terms of these equalities we must also add the helicopter parasite drag force moment, which will generally be a climbing moment.

Helicopter transverse equilibrium is that state of the helicopter in which there is no rotation about the longitudinal axis. The transverse equilibrium conditions are expressed in general form by the following equalities: $\Sigma F_z = 0$, no side displacement, this equality expresses the first transverse equilibrium characteristic; $\Sigma M_x = 0$, no rotation about the longi- /156
tudinal axis, this equality expresses the second transverse equilibrium characteristic.

In order to express the transverse equilibrium conditions through the forces acting on the helicopter during hovering and through the transverse moments of these forces, we must examine the conditions for equilibrium of the single-rotor helicopter without an aft fin and the single-rotor helicopter with an aft fin.

In the single-rotor helicopter without an aft fin, the tail rotor is located right on the tail boom (Figure 97a).

This helicopter can have transverse equilibrium in the hover regime only in the presence of a bank in the direction opposite the tail rotor thrust (Figure 97c). A side component of the weight force $G_2 = G \sin \gamma$ (γ is the

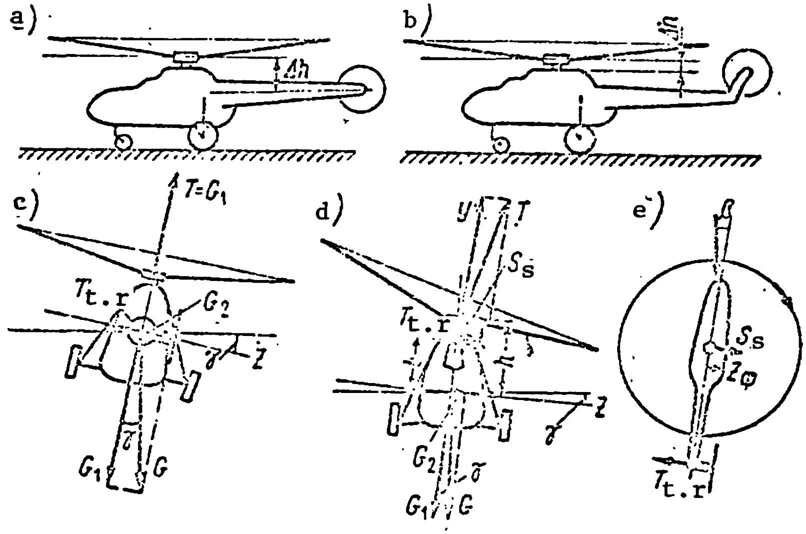

Figure 97. Helicopter lateral equilibrium.

bank angle) is formed when the helicopter banks. This component balances the tail rotor thrust force. Then the equality expressing the first transverse equilibrium characteristic is written as follows

$$T_{t.r} = G_2 = G \sin \gamma \text{ or } T_{t.r} - G_2 = 0.$$

Hence we find the magnitude of the bank angle

$$\gamma = \arcsin \frac{T_{t.r}}{G}.$$

The thrust force vector is not deflected from the helicopter symmetry plane, therefore the thrust force moment about the helicopter longitudinal axis will be zero. The moment of the horizontal hinges will also be zero. The tail rotor thrust force is applied to the longitudinal axis of the /157 helicopter, and its moment will also be zero. Therefore, the second helicopter equilibrium characteristic will be expressed in general form by the equality $\Sigma M_x = 0$.

The transverse equilibrium of this type of helicopter in the other flight regimes can be achieved in the same way as in the hover regime, i.e., by banking; or it can be obtained by sideslipping the helicopter in the direction of the tail rotor thrust (Figure 97d). During the sideslip there is formed the side air pressure force Z_f on the fuselage, and this force balances the tail rotor thrust force

$$T_{t.r} - Z_{\phi} = 0.$$

The advantages of the helicopter of this type are: lower loading on the tail boom, since there is no twisting moment of the tail rotor thrust force, and lower helicopter weight in view of absence of the vertical fin. The disadvantages of the helicopter include:

(1) large bank angle during hover, which creates discomfort for passengers and crew and makes helicopter control difficult;

(2) large bank angle or large sideslip in translational flight, which increases helicopter parasite drag;

(3) probability of damage to the tail rotor in view of its very low position;

(4) danger to servicing personnel because of the low tail rotor location.

Therefore, helicopters without a vertical fin are seldom encountered.

The presence of a vertical fin on which the tail rotor is mounted eliminates these drawbacks, since the tail rotor is raised above the ground and its axis approaches the main rotor plane of rotation (Figure 97b). Let us examine the transverse equilibrium of a helicopter with vertical fin (Figure 97d). This helicopter, like the helicopter without a vertical fin, hovers with a bank angle, but the bank angle will be small, about 1°, and is not noticeable

in practice. The necessity for the bank angle follows from the transverse equilibrium conditions. The bank angle leads to the side component G_2 - G sin γ of the weight force. The first transverse equilibrium criterion will be expressed by the equatior

$$T_{t.r} = S_s + G_2 \text{ or } T_{t.r} - (S_s + G_2) = 0.$$

To satisfy this condition the main rotor thrust force vector (coning axis) must be deflected by some angle in the direction opposite the tail rotor thrust direction; then the side component S_s is created, which together with the force G_2 balances the tail rotor thrust.

Since the tail rotor is raised above the helicopter longitudinal axis, the transverse moment rolling $M_{T_{t.r}} = T_{t.r}b$ is developed. This moment is balanced by the side force moment $M_s = S_s h$ and the moment of the horizontal hinges relative to the longitudinal axis. Consequently, the second transverse equilibrium criterion will have the form /158

$$T_{t.r}b = S_s h + M_{hh_x}.$$

Why does the helicopter require a bank angle in hover?

If there were no bank angle,the first transverse equilibrium criterion could be written as

$$T_{t.r} = S_s \text{ or } T_{t.r} - S_s = 0.$$

But to obtain the side force S_s the main rotor cone of rotation must be tilted, which leads to the appearance of a moment of the horizontal hinges and a tail rotor thrust moment. If h - b, then

$$T_{t.r} = \frac{S_s h + M_{hh_x}}{b} = S_s + \frac{M_{hh_x}}{b}$$

i.e., $T_{t.r} > S_s$ by $\frac{M_{hh_x}}{b}$.

But $T_{t.r} = S_s$; therefore contradictory requirements are obtained: the tail rotor thrust must at the same time be larger than and equal to the side force, which is not possible; therefore a small bank angle and the side weight force G_2 are required for equilibrium.

During flight with translational velocity, transverse equilibrium is achieved either as a result of a bank angle, as in hovering, or as the result of sideslip, which results in side pressure force on the fuselage. The transverse equilibrium condition will be expressed by the equalities

$$T_{t.r} = S_s + Z_\phi \text{ and } T_{t.r}b = S_s h + M_{hh_x}.$$

Helicopter directional equilibrium is the state of the helicopter in which it does not rotate about the vertical axis. The directional equilibrium conditions are

$$\sum F_{zy} = 0; \quad \sum M_y = 0.$$

To ensure the latter condition it is necessary that the reactive moment be balanced by the moments of tail rotor thrust and main rotor thrust side force. Then

$$M_r = M_{T_{t.r}} = T_{t.r} L_{t.r}.$$

Here the following circumstance must be noted. With change of the main rotor thrust there will also be a change of its reactive moment, i.e., the directional equilibrium will be disturbed. Therefore, when the main rotor thrust is changed, it is necessary to change the tail rotor thrust to preserve directional equilibrium. But this requirement complicates the control helicopter.

In the translational flight regimes with horizontal velocity, directional equilibrium is achieved in the same way as in the hovering regime /159

$$M_r = T_{t.r} L_{t.r} \pm Z_{\phi} b,$$

where b is the distance from the point of application of the force Z_f to the helicopter vertical axis.

§ 71. Helicopter Static Stability

General analysis of static stability. Static stability is the capability of the helicopter to restore disturbed equilibrium by itself after removal of the factors causing this disturbance. Static stability is attitude stability. The helicopter will be stable if after equilibrium is disrupted stabilizing moments develop on the helicopter, i.e., moments directed to restore the previous attitude. Static stability is amplified as a result of damping moments. A damping moment is one which is directed opposite the oscillatory motion of the helicopter about some axis. The difference between the stabilizing and damping moments is that the former arises as a result of equilibrium disruption and acts after the termination of this disruption. The damping moment acts only in the process of equilibrium disruption and is directed opposite the deviation.

If during and after equilibrium disruption, moments appear on the helicopter which deflect it still further from the previous attitude, such moments are termed destabilizing. The helicopter on which destabilizing moments arise is termed statically unstable. The helicopter on which no moments arise during and after the equilibrium disruption process has indifferent equilibrium and is termed statically neutral.

The factors which disturb equilibrium are:

(1) unsettled state of the air ("turbulence");

(2) random deflection of the control levers;

(3) failure of some part of the helicopter;

(4) change of the cg location.

Characteristic of helicopter equilibrium is the intimate interconnection of the individual equilibrium modes with one another. For example, if longitudinal equilibrium is disrupted, i.e., if the helicopter rotates about the transverse axis, the main rotor angle of attack changes. This involves a change of the thrust force and the reactive moment of the main rotor. The change of the reactive moment disrupts the directional equilibrium. Disruption of directional equilibrium leads to change of the tail rotor thrust force /160
and change of the moment of this force relative to the longitudinal axis, which means disruption of the helicopter transverse equilibrium. The intimate interconnection among the equilibrium modes requires constant action from the pilot, directed toward restoring the disrupted equilibrium, i.e., helicopter control becomes more complicated. Does the helicopter have static stability? In order to answer this question we must examine the static stability of the main rotor, the static stability of the fuselage, and the effect of the stabilizer and tail rotor on the static stability.

Main rotor static stability with respect to velocity. When the helicopter equilibrium is disrupted, two motion parameters of the main rotor change: the flight velocity and the angle of attack. Let us assume that the helicopter is performing horizontal flight at the velocity V (Figure 98a). For some reason the flight velocity is increased by ΔV. As a result there is an increase of the flapping motions of the blades, the main rotor cone axis is deflected aft from the previous position, which is shown dashed in the figure, by the angle ε. The tilt of the coning axis leads to the appearance of the force P_x, directed opposite the flight direction. Under the influence of this force the main rotor velocity will decrease.

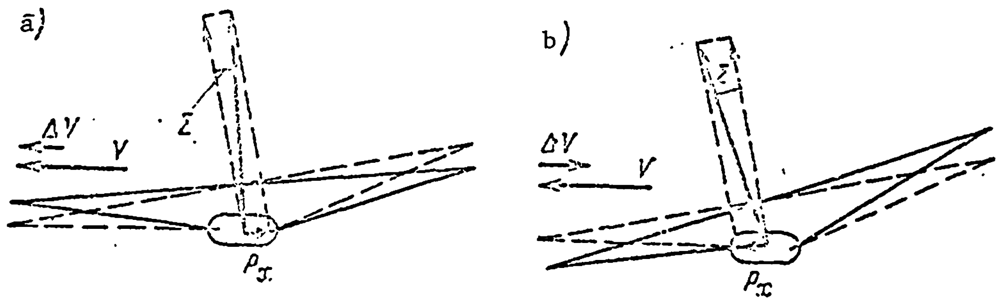

Figure 98. Main rotor speed stability.

If the flight velocity is reduced by ΔV (Figure 98b), the cone axis is deflected forward, the force P_x in the direction of flight develops, and the flight velocity will increase. The conclusion is that the main rotor has static stability with respect to velocity.

Main rotor static stability with respect to angle of attack. The helicopter is flying horizontally and the main rotor angle of attack is A. Under the influence of a vertical air current, the helicopter drops its nose and the main rotor angle of attack is reduced by ΔA (Figure 99a). Prior to disruption of equilibrium, the main rotor thrust force vector passed through the helicopter center of gravity and the moment of the thrust force was zero. Upon disruption of the equilibrium, the thrust force vector T is deflected forward and the moment $M_z = T_1a$ develops relative to the helicopter transverse axis; this moment rotates the helicopter and the main rotor in the direction to reduce the angle of attack. Consequently, this moment will be destabilizing.

If for some reason the main rotor angle of attack is increased (Figure 99b), the thrust force vector tilts aft and a nose-up moment $M_z = T_1a$ is created, which causes the angle of attack to increase. A destabilizing thrust moment is created. The conclusion is that the main rotor is unstable with respect to angle of attack. /161

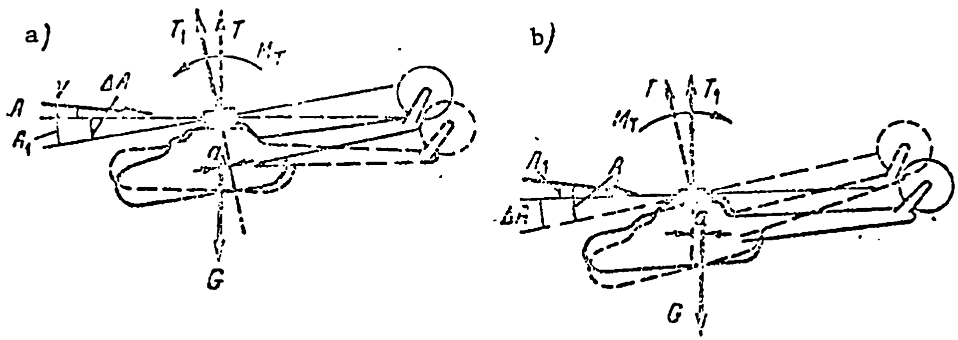

Figure 99. Main rotor angle-of-attack stability.

Helicopter fuselage static stability. The fuselage has the second largest influence, after the main rotor, on the static stability of the helicopter. The fuselage of the single-rotor helicopter has static instability about all three axes. A small stabilizer is installed at the aft end of the fuselage to improve longitudinal static stability in horizontal flight. In the hover regime and in flight at low velocity the stabilizer has practically no effect on longitudinal static stability. But with increase of the flight velocity and with reduction of the angle of attack,the longitudinal instability present in a hover decreases, and at negative angles of attack the fuselage plus stabilizer has longitudinal static stability. Thus, the fuselage of the Mi-1 has static stability at angles of attack from -10 to -2°. At positive angles of attack,the fuselage has indifferent longitudinal equilibrium.

The helicopter stabilizer is controllable. Stabilizer control is accomplished with the aid of the collective-throttle lever. When this lever is moved upward,main rotor pitch and stabilizer incidence angle are increased. The controllable stabilizer makes it possible to tilt the helicopter fuselage to a negative angle at the maximal slight speed. If the stabilizer angle were constant, a large negative lift force would develop on the stabilizer at a negative angle of attack, and this force would create a large nose-up moment to prevent tilting of the fuselage. When the rotor pitch is decreased the

stabilizer incidence angle becomes negative and a nose-up moment is developed, which aids in transitioning the main rotor into the autorotative regime. The main rotor angle of attack becomes positive, and as a result of the increase of the angle of each blade element the rotor rpm increases. /162

The tail rotor also affects the fuselage static stability. The fuselage acquires directional stability as a result of the tail rotor. Thus, when directional equilibrium is disrupted, if the helicopter turns to the right for example (with right-hand rotation of the main rotor), the angles of attack of the tail rotor blade elements increase, and the tail rotor thrust force increases by ΔT. The moment of the tail rotor thrust also increases and equilibrium is restored.

If the helicopter turns to the right, the blade element angles of attack will decrease and therefore the thrust will decrease. The tail rotor moment becomes less than the main rotor reactive moment, and this leads to restoration of the equilibrium.

Since the tail rotor is mounted above the helicopter longitudinal axis and creates a transverse thrust moment, this leads to increase of the transverse static stability. Therefore, the tail rotor gives the fuselage directional and transverse static stability.

The conclusion is that the helicopter has slight static stability in horizontal flight and indifferent equilibrium in hover and in the other vertical flight regimes.

The static stability of twin-rotor helicopters differs somewhat from that of the single-rotor helicopter. The two-rotor helicopter with tandem arrangement of the rotors has considerably greater longitudinal static stability, while the two-rotor helicopter with side-by-side arrangement has higher transverse stability. This is explained by the variation of the thrust of the lifting rotors when equilibrium is disrupted.

Effect on helicopter static stability of horizontal hinge offset. If the main rotor hub has offset horizontal hinges, the horizontal hinge moments have considerable effect on the longitudinal and transverse static stability of the helicopter. The larger the horizontal hinge offset and the higher the main rotor rpm, the larger the main rotor damping moment and the greater the helicopter static stability. Thus, increase of the static stability is achieved by increasing the horizontal hinge offset. The appearance of the damping moment is explained by the gyroscopic properties of the main rotor.

As is well known, the basic property of the gyroscope is its ability to maintain the attitude of its axis of rotation fixed in space. This property shows up more strongly, the larger the mass and the higher the rpm of the rotating body. What effect does the gyroscopic effect of the main rotor have on the behavior of the helicopter? We shall use an example to investigat this question.

Let us assume that the longitudinal equilibrium of the helicopter has been disturbed and it has started to rotate about the transverse axis in the nose- /163 down direction (Figure 100). In view of its gyroscopic properties the main rotor will lag behind the helicopter rotation, therefore the main rotor hub axis is deflected from the cone-of-rotation axis. The horizontal hinge moment M_{hh} = Nc will be directed opposite the tilt of the helicopter; therefore it will be a damping moment. When equilibrium is disrupted, the helicopter will rotate relative to the stationary cone-of-rotation axis of the main rotor as a result of the gyroscopic effect. And this means that the helicopter center of gravity will also displace together with the helicopter

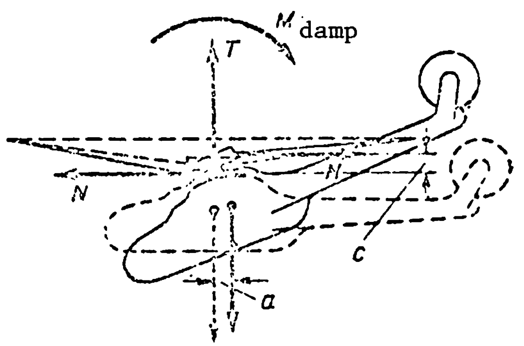

Figure 100. Main rotor damping moment.

relative to the cone-of-rotation axis (shown dashed in Figure 100). The result is the formation of the thrust moment $M_T = Ta$. The damping moment will be equal to the sum of the two moments $M_{damp} = M_{hh} + M_T = Nc + Ta$. The damping moment will be larger, the higher the main rotor rpm, the larger the horizontal hinge offset, and the lower the position of the helicopter center of gravity, i.e., the larger the vertical cg displacement. The arm a and the thrust force moment will increase with increase of the distance Y from the hub rotation plane to the center of gravity.

§ 72. Helicopter Dynamic Stability

General analysis of dynamic stability. While static stability defines the attitude stability, dynamic stability defines the nature of the helicopter motion after disruption of equilibrium. In equilibrium the helicopter travels in a straight line with constant velocity and without rotation. Such motion is called undisturbed. If equilibrium is disrupted, the helicopter rotates about its axes and the flight velocity and direction change. This motion is called disturbed. Disturbed motion may be either aperiodic or oscillatory.

Aperiodic motion is motion in one direction from the equilibrium position. For example, when equilibrium is disturbed the helicopter center of gravity deviates (Figure 101, solid line). After elimination of the factor causing disruption of the equilibrium, the nature of the disturbed motion may differ.

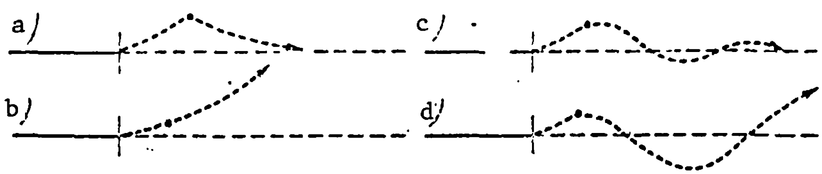

Figure 101. Dynamic stability.

If the center of gravity approaches the line of unperturbed motion (Figure 101a, dashed line), the helicopter has aperiodic stability; if the /164

helicopter center of gravity continues to deviate further from the equilibrium line (Figure 101b) the helicopter has aperiodic instability. Oscillatory motion is periodic back-and-forth motion relative to the equilibrium line. If after disruption of equilibrium the helicopter center of gravity travels along a wave-like curvilinear trajectory and this motion is damped, the helicopter has oscillatory dynamic stability (Figure 101c). If the amplitude of the disturbed oscillatory motion increases, the helicopter has dynamic instability or oscillatory instability (Figure 101d).

Most frequently the disturbed motion of the helicopter is oscillatory, and the oscillations will be complex, since the helicopter oscillates simultaneously about all axes. Moreover, the short and long periodic oscillations are superposed on one another. The short-period helicopter oscillations are those about the center of gravity with account for the influence of the main rotor damping moment; the long-period oscillations are those about a center located at a considerable distance from the helicopter.

Helicopter transverse oscillations in the hovering regime. Let us assume that the helicopter banks to the angle γ in the hovering regime (Figure 102 a). We resolve the helicopter weight force into the components: G_1 acting in the plane of symmetry, and G_2 perpendicular to this plane. The force $G_2 = G \sin \gamma$ is unbalanced and causes sideslip of the helicopter. As a result of the increase of the sideslip velocity,the main rotor cone-of-revolution axis will tilt to the side opposite the slip (Figure 102b). The force P_x is created, which reduces the sideslip velocity and the moment of this force reduces the bank angle. But the force P_x is less than the force G_2; therefore, the sidelsip velocity will increase and the velocity will be maximal at the moment the helicopter arrives at the position shown in Figure 102c.

The helicopter continues its motion in the same direction (Figure 102d). Then the force G_2 changes from driving to retarding, and the sideslip velocity

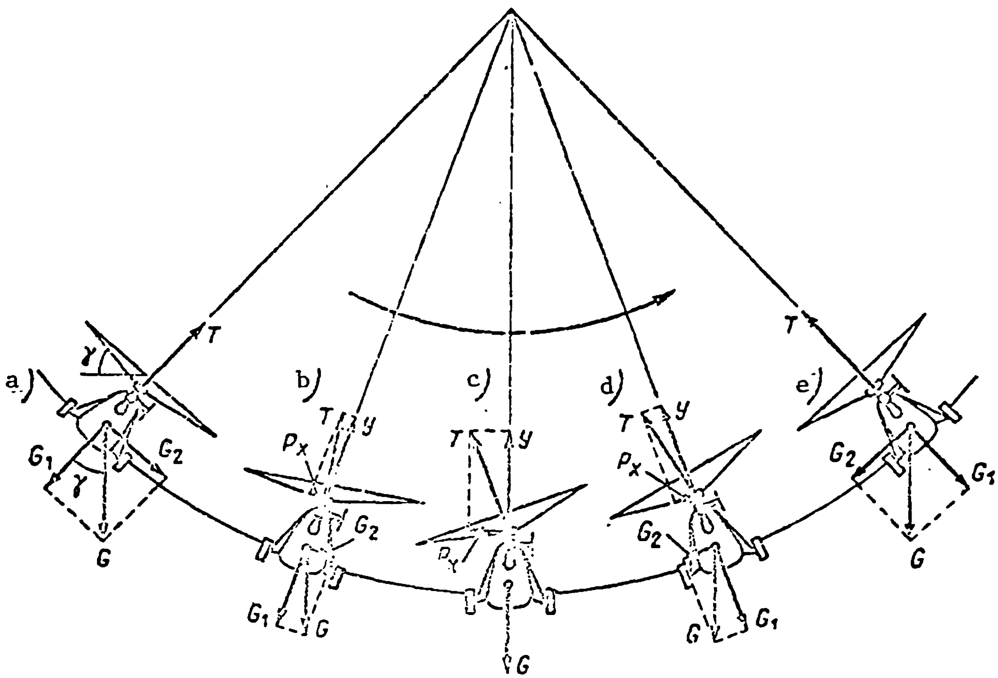

Figure 102. Helicopter lateral oscillations.

decreases. As a result, the tilt of the cone-of-revolution axis decreases and the moment of the force P_x about the longitudinal axis banks the helicopter in the opposite direction. When the helicopter reaches its maximal /165 deviation (Figure 102e) further motion terminates. The cone-of-revolution axis coincides with the hub axis and the force $P_x = 0$. But the force G_2 reaches a maximum and causes motion in the reverse direction and the whole cycle repeats. This transverse rocking of the helicopter will increase continuously, and the helicopter will turn over if these oscillations are not terminated in time.

We have examined in this example only the transverse oscillations, but in reality the transverse oscillations are supplemented by longitudinal and directional oscillations; therefore the pattern of the oscillatory motions will be more complex.

Longitudinal oscillations of helicopter in flight with horizontal velocity. If the longitudinal equilibrium of a helicopter is disturbed, longitudinal oscillations develop (Figure 103), i.e., the helicopter will travel along a wave-like trajectory. The existence of such oscillations is confirmed by flight tests in which automatic instruments record the nature of the helicopter oscillations about all axes. The longitudinal oscillations /166 have a considerably longer period (total oscillation time) than the transverse oscillations. The amplitude of the longitudinal oscillations increases in the course of time, although more slowly than the amplitude of the transverse oscillations. Helicopter oscillations about the vertical axis also take place; however, they are performed with a period longer than the transverse oscillations but shorter than the longitudinal oscillations.

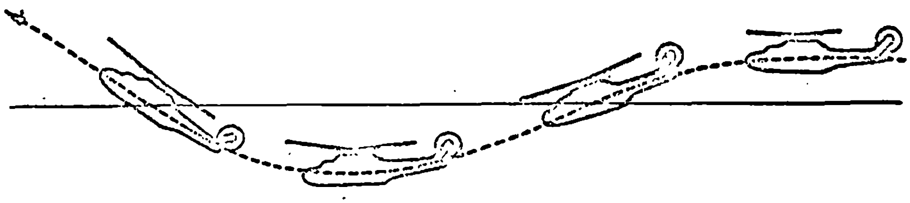

Figure 103. Helicopter longitudinal oscillations.

From this analysis we can conclude that the helicopter has dynamic instability. Therefore, if the equilibrium of the helicopter is disturbed it will have an oscillatory motion with increasing amplitude and cannot by itself eliminate these oscillations. This means that in every case of equilibrium disruption the pilot must take measures to restore the equilibrium, i.e., he must control the helicopter.

§ 73. Concept of Helicopter Control

By the term "helicopter control" we mean the actions of the pilot directed toward achieving two objectives: restoration of disturbed equilibrium and disruption of existing equilibrium. We see that these objectives are

contradictory, but in combination they lead to achievement of the flight objectives.

The actions directed toward restoring disrupted equilibrium of the helicopter are necessary because flight takes place most frequently in rough air, when there is continuous disruption of equilibrium and it must be continuously restored. Otherwise the helicopter cannot fly in the required direction and with the required velocity. The work done by the pilot to restore equilibrium is the primary control work. This operation has recently been mechanized with the aid of autopilots. Helicopter equilibrium is restored by the action of the control moments about the principal axes of the helicopter.

The control moments are created by the main and tail rotor thrust forces. This means that the helicopter control organs are the main and tail rotors. The pilot's actions directed toward disruption of helicopter equilibrium are required when it is necessary to alter the direction or velocity, i.e., alter the flight regime. To change the flight regime it is necessary to change the magnitude and direction of the main rotor thrust force and change the attitude of the helicopter in space, which is achieved by the action of the control moments created by the main and tail rotor thrust forces. This means that in the final analysis helicopter control reduces to control of the main rotor thrust force vector and control of the magnitude of the tail rotor thrust. The magnitude of the main rotor thrust force vector is changed by changing the collective pitch; the direction of this force vector is changed by changing the main rotor cyclic pitch.

§ 74. Change of Main Rotor Collective and Cyclic Pitch

Simultaneous rotation of all the blades relative to the axial hinges in the same direction and through the same angle is termed collective pitch change. Increase of the collective pitch leads to increase of the main rotor thrust force. Sequential change of the blade pitch in azimuth is termed cyclic /167

pitch change. During cyclic pitch change,the pitch of each blade increases over a 180° azimuth range and decreases in the other half of the circle (Figure 104a).

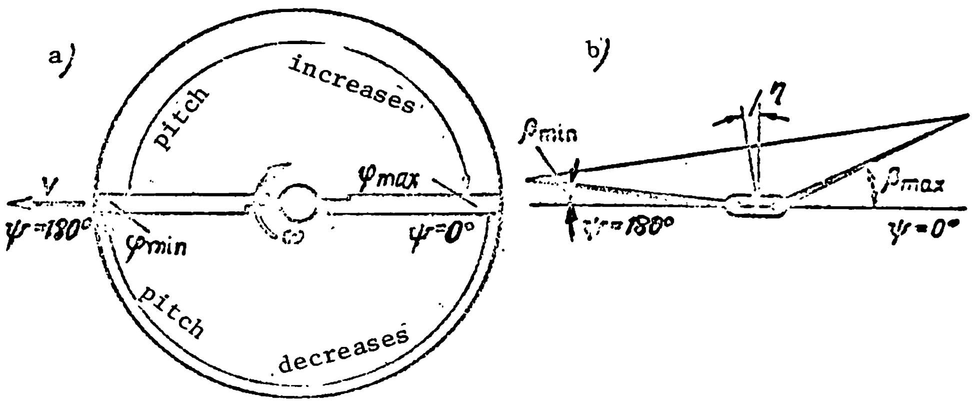

Figure 104. Main rotor cyclic pitch variation.

When the pitch is changed,the blade thrust changes and its moment about the horizontal hinge changes, which leads to flapping motions and tilt of the cone-of-revolution axis and deflection of the thrust force vector.

If the thrust force vector needs to be deflected in the direction of the 210° azimuth, the blade pitch must be minimal at this aximuth and maximal at the opposite azimuth — 30°. Then the pitch decreases from the 30° azimuth to the 210° azimuth and increases from the 210° azimuth to the 30° azimuth. A similar pitch change is observed for each blade. Change of the collective and cyclic pitch is accomplished with the aid of a special system — the main rotor tilt control.

§ 75. Purpose and Principle of the Main Rotor Tilt Control System

The main rotor tilt control is designed to control the collective and

cyclic pitch. This system is used to control the main rotor thrust force in magnitude and direction. Therefore, the tilt control is the most important unit of the helicopter control system. There are three types of main rotor tilt controls: ring, "spider," and crank types. The latter type of tilt control is used only on two-rotor helicopters with side-by-side positioning of the rotors. /168

The ring type tilt control can be used on all helicopters. It includes (Figure 105):

(1) movable ring (plate);

(2) fixed ring;

(3) slider;

(4) universal hinge or cardan;

(5) scissors or bellcrank;

(6) vertical control rods.

The movable ring of the tilt control rotates relative to the fixed ring. It is driven from the main rotor hub by means of a scissors. On the movable ring there are levers which are connected with the blade pitch control horns by means of vertical links. The fixed ring is connected with the slider by means of a universal, which consists of a ring and two mutually perpendicular shafts. The universal permits the tilt control rings to deflect in any direction. If the plane of the rings is perpendicular to the main rotor shaft /169
axis, when the movable ring rotates the vertical links will not have any vertical displacement and the blade pitch will not change. Consequently, in this case the rotor will not have any cyclic pitch change.

If the plane of the rings is tilted forward, the vertical links will be at the lowest position at the 180° azimuth and the blades will have the minimal pitch at this azimuth. At the 360° azimuth the links occupy the highest

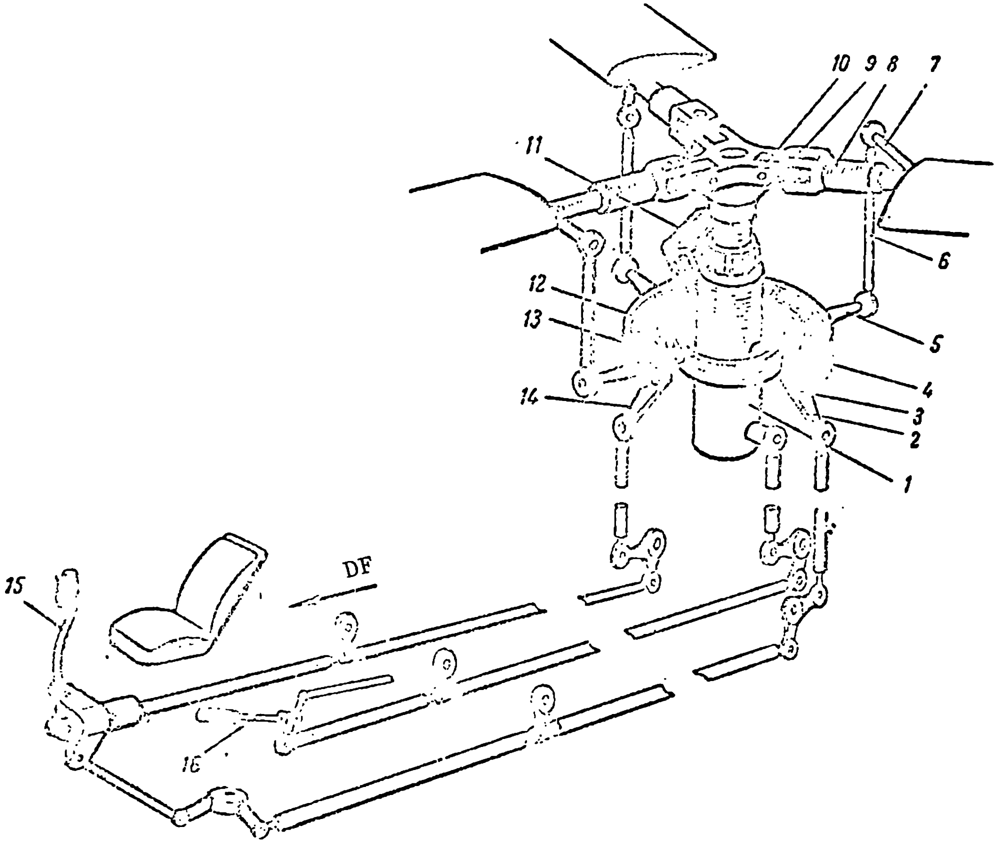

Figure 105. Ring-type tilt control: 1 - slider; 2 - lateral control lever; 3 - fixed ring; 4 - movable ring (plate); 5 - lever on movable ring; 6 - vertical link; 7 - blade pitch change horn; 8 - axial hinge; 9 - vertical hinge; 10 - horizontal hinge; 11 - scissors; 12 - universal ring; 13 - universal longitudinal axis; 14 - longitudinal control lever; 15 - cyclic pitch lever; 16 - collective-throttle lever.

position and the blade pitch will be maximal. From 0 to 180° azimuth the links move downward and the blade pitch decreases. From 180 to 360° azimuth the links move upward and the pitch increases. Cyclic pitch change is accomplished in this way. We mentioned previously that as a result of cyclic pitch change the cone-of-revolution axis is tilted in the direction of minimal pitch. This means that, in order to tilt the cone axis in any direction, the plane of rotation of the movable ring of the tilt control must be tilted in

this same direction. The tilting is accomplished with the aid of two levers on the fixed tilt control ring. The control system levers are connected with the cyclic pitch stick, located in the cockpit. When the stick is moved forward, the motion is transmitted to the longitudinal control lever on the fixed ring of the tilt control, and the fixed ring rotates around the transverse axis of the universal so that its leading edge descends.

This means that the pitch is minimal at the 180° azimuth, and the cone-of-revolution axis is tilted forward. When the cyclic pitch stick is moved aft, the leading edge of the tilt control ring rises and the pitch is minimal at the 0° azimuth and the cone-of-revolution axis tilts aft. The conclusion is that when the stick is deflected the cone axis tilts in the same direction. When the stick is moved to the right or left the motions are transmitted from the stick to the lateral control lever, and the tilt control ring is rotated about the longitudinal axis of the universal. In this case our previous conclusion still holds: when the stick is deflected the main rotor cone-of-revolution axis deflects in the same direction. Control of the cyclic pitch and direction of the thrust force vector is accomplished from the cockpit with the aid of the cyclic pitch stick.

On the tilt control slider, there is a lever which is connected by the control system with the collective-throttle lever located in the cockpit. When the collective-throttle lever is moved up, the tilt control slider rises. All the vertical links move upward together and rotate all the blades to a higher incidence angle. In this way the collective pitch is increased. When the collective-throttle lever is moved down, the tilt control slider lowers and the main rotor collective pitch decreases. The tilt control slider is connected by the control linkage with the stabilizer rotation lever. Therefore, change of the collective pitch is associated with change of the stabil- /170
izer incidence angle. On some helicopters the main rotor collective pitch control is coupled with the tail rotor pitch control.

When this coupling is used on helicopters with right-hand rotation of the main rotor, increases of the collective thrust leads to increase of the pitch and of the tail rotor thrust force, and control of the helicopter is made easier. With increase of the main rotor collective pitch there is an increase of its reactive moment, which causes the helicopter to turn to the left. This rotation is compensated by increase of the tail rotor thrust.

We have examined a very simple tilt control scheme of the ring type. The real tilt control has two essential characteristics which must be mentioned. The first characteristic is that the hinges of all the levers on the tilt control movable and fixed rings are located in the same plane, which passes through the point of intersection of the universal axes. This arrangement of the hinges makes possible independence of the action of the longitudinal and lateral control of the helicopter. The second characteristic amounts to the following.

In the functional schematic (Figure 106) the longitudinal and lateral control levers are located on the fixed ring opposite the universal axes. In the real tilt control these levers are located at some angle relative to the universal axes, which is called the control lead angle (χ). In the absence of lead the cone axis will not deflect in the direction of cyclic control stick deflection, rather at some angle ahead in the direction of rotor rotation. This lag in the deflection of the cone-of-revolution axis is associated with the inertia of the blades.

At the 180° azimuth the pitch is minimal, but the blade flapping angle will not be minimal, since the blades will continue to flap down by inertia. This means that the cone-of-revolution axis tilts in the direction of the 210° azimuth rather than in the direction of the 180° azimuth, i.e., the deflection of the cone-of-revolution axis will not coincide with the stick deflection, and this makes control of the helicopter more difficult. Therefore, the tilt control ring is deflected with a lead angle $\chi = 25 - 30°$, which then leads to coincidence of the deflection of the stick and the main rotor cone-of-revolution axis.

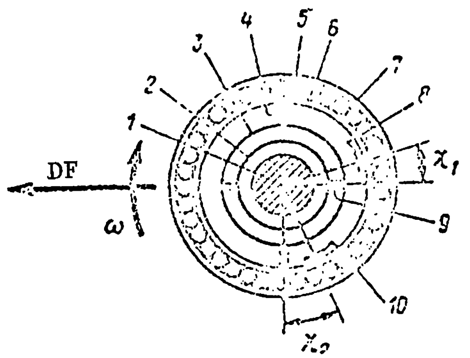

Figure 106. Main rotor control lead: 1 - main rotor shaft; 2 - slider; 3 - universal ring; 4 - universal lateral axis; 5 - tilt control fixed ring; 6 - bearing balls; 7 - tilt control movable ring; 8 - longitudinal control lever; 9 - universal longitudinal axis; 10 - lateral control lever; χ_1 - longitudinal control lead angle; χ_2 - lateral control lead angle.

§ 76. Single-rotor Helicopter Control Principles

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Helicopter control involves control of the rotation of the helicopter about its principal axes and control of the vertical displacement. On this basis the entire control complex considered as the sum of the pilot's actions can be divided into longitudinal, lateral, and directional control, and also control of the up-and-down displacement of the helicopter. This division is purely arbitrary, since the pilot's control actions are unified and are accomplished simultaneously and synchronously. However, this division facilitates study of the control question and corresponds to the construction of the control system, which includes four control loops which are independent of one another and are named the same as the names of the particular control modes.

Lontitudinal control is control of helicopter rotation about the transverse axis. It is achieved by the action on the helicopter of the longitudinal control moments $M_{z_{cont}}$, which are created with fore-and-aft deflection of the cyclic pitch control stick. As a result of the tilt of the cone-of-revolution axis in the direction of the stick deflection, the main rotor thrust vector tilts. If prior to deflection of the stick the helicopter was in equilibrium, i.e., the thrust force moment was zero (Figure 107a), after forward deflection of the stick the cone-of-revolution axis deflects in the same direction. The thrust force vector passes at the distance a from the transverse axis and creates the thrust moment M_T = Ta, which will be a diving moment. In addition

to the thrust moment, the horizontal hinge diving moment M_{hh_z} = Nc is created. The sum of these two moments creates the longitudinal control moment. The helicopter will be rotated about the transverse axis in the nose-down direction under the action of this moment. Aft deflection of the stick leads to the formation of a nose-up pitching moment, under the action of which the helicopter nose rises.

If the helicopter has longitudinal static stability, the rotation will continue until the stabilizer longitudinal moment balances the control moment. If the helicopter does not have static stability, it will be necessary to deflect the stick in the opposite direction to stop the rotation. As a result of helicopter rotation there is a change of the inclination of the thrust force vector and its horizontal component P. This leads to a change of the flight velocity. Hence we can conclude that forward deflection of the cyclic pitch stick leads to lowering of the helicopter nose and increase of the flight speed. Aft deflection of the stick leads to the helicopter nose rising and reduction of the flight speed. If the stick is moved aft while the helicopter is hovering, it will start to move backward. Therefore the operation of the cyclic pitch stick is analogous to the operation of the control stick in an airplane. /172

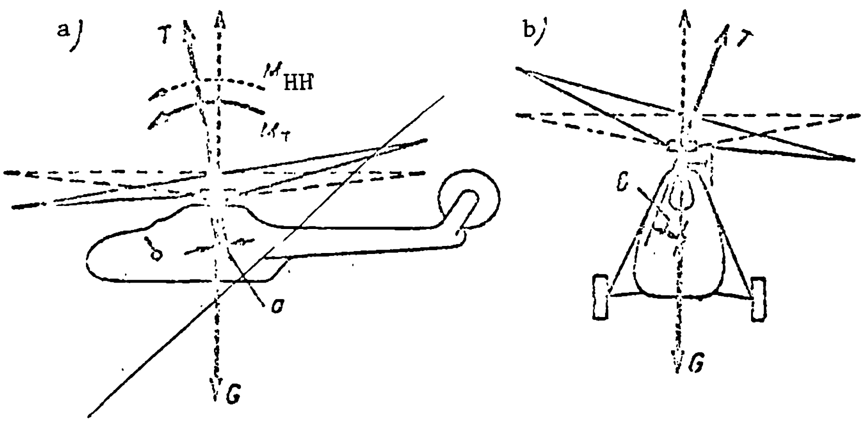

Figure 107. Control of single-rotor helicopter.

Lateral control refers to control of helicopter rotation about the longitudinal axis. Lateral control is accomplished by deflecting the cyclic pitch stick to the right or left. The main rotor coning axis and the thrust force vector tilt to the same side as the stick (Figure 107b). The deflection of the thrust force vector and the main rotor plane of rotation leads to creation of the lateral control moment as the sum of the moments of the thrust force and the horizontal hinges. The magnitude of the moment will be larger, the higher the main rotor rpm, the larger the stick deflection, and the lower the position of the helicopter center of gravity. Under the influence of the control moment, the helicopter will rotate until the stick is moved in the opposite direction.

Directional control refers to control of helicopter rotation about the vertical axis. The helicopter rotates about the vertical axis under the influence of the directional moment, which is created as a result of the difference of the main rotor reactive moment and the tail rotor thrust moment $M_{y_{cont}} = M_r - M_{T_{t.r}}$. Change of the tail rotor thrust force and its moment about the helicopter vertical axis is accomplished by deflecting the directional control pedals. These pedals are coupled by linkage with the pitch change mechanism mounted on the tail rotor gearbox.

If the right pedal is pushed, the tail rotor pitch is increased. As a result of increase of the thrust force by the amount ΔT, its moment increases. The tail rotor thrust moment becomes greater than the main rotor reactive moment and the helicopter turns to the right. If the left pedal is pushed, the tail rotor pitch is reduced. In view of the decrease of the tail rotor thrust and moment,the latter becomes smaller than the main rotor reactive moment. Under the influence of this moment the helicopter turns to the left.

§ 77. Dual-Rotor Helicopter Control Principles /173

Control principle of dual-rotor helicopter with tandem arrangement of the lifting rotors. Longitudinal control of the helicopter is achieved by

deflecting the stick fore and aft. This leads to cyclic change of the pitch of the lifting rotors, as a result of which the axes of the cones-of-rotation are tilted forward or backward, i.e., in the direction of the stick (Figure 108). In addition to the cyclic variation of the pitch there is a differential change of the collective pitch, in which the thrust of one rotor is increased while that of the other is decreased.

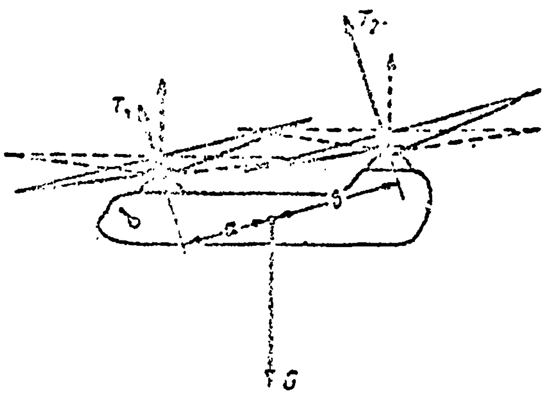

Figure 108. Control of tandem twin-rotor helicopter.

If the stick is deflected forward, the axes of the cones-of-rotation of the lifting rotors are tilted forward. The collective pitch of the front rotor is reduced and that of the aft rotor is increased. As the thrust force vectors tilt, there is a change of the thrust force arms relative to the helicopter transverse axis. The result is the creation of a diving moment equal to the difference of the thrust moments of the front and rear rotors.

$$M_{x_{cont}} = T_1 a - T_2 b.$$

Under the influence of this moment the helicopter nose will drop, increasing the flight speed. If the stick is moved aft, a climbing moment is created and the helicopter nose will rise, reducing the flight speed.

Lateral control of the helicopter is achieved by deflecting the stick to the right and left. This leads to simultaneous identical change of the cyclic pitch of the front and rear lifting rotors. A lateral control moment appears which then causes rotation of the helicopter around the longitudinal axis.

Directional control, or control of helicopter rotation around the vertical axis, is accomplished with the aid of the directional control pedals. Deflection of the pedals leads to differential change of the cyclic pitch of the lifting rotors. The axes of the cones-of-rotation deflect in opposite directions, forming the directional control moment as a result of side forces. If the right pedal is pushed, the coning axis of the front rotor is deflected to the left. The side components of the lifting rotor thrust forces create /174 a pair, whose moment turns the helicopter to the right.

Control principle of the dual-rotor helicopter with side-by-side lifting rotors. The control stick is moved fore and aft for longitudinal control, and this causes the same change of the cyclic pitch of the lifting rotors and deflection of the axes of the cones-of-revolution in the direction of stick displacement. This creates a longitudinal control moment (just as in the case of the single-rotor helicopter).

Lateral control is accomplished by deflecting the stick to the right or left. This deflection leads to differential change of the collective pitch of the lifting rotors. If the stick is moved to the right,the collective pitch of the right rotor is reduced and that of the left rotor is increased. Change of the collective pitch causes change of the thrust forces. The difference of the thrust forces of the right and left lifting rotors leads to the creation of a lateral moment which then causes a bank to the right. The helicopter with side-by-side arrangement of the lifting rotors has an auxiliary wing which gives the helicopter lateral stability.

Directional control of the helicopter is accomplished by the control moment which is created by differential change of the cyclic pitch and tilting of the axes of the cones-of-rotation in opposite directions: forward and backward. If the right pedal is pushed, the axis of the cone-of-rotation of the right rotor is deflected aft while that of the left is deflected forward. The horizontal components of the thrust forces create the directional control moment which causes rotation of the helicopter to the right.

Control principle of dual-rotor helicopter with coaxial rotors. Longitudinal and lateral control is accomplished similarly to the control of the single-rotor helicopter, i.e., by cyclic change of the pitch of the upper and lower lifting rotors. When the stick is deflected, the axes of the cones-of-rotation deflect in the same direction as the stick, creating the longitudinal or lateral control moment.

Directional control is accomplished by deflection of the pedals, which leads to differential change of the collective pitch of the lifting rotors. This does not cause any change of the overall thrust, but leads to a change of the reactive moments of the lifting rotors and the helicopter turns in the direction of the action of the larger reactive moment.

Control of the vertical displacement of all helicopters is the same.

When the collective-throttle lever is moved up, the collective pitch of all the lifting rotors is increased, which leads to increase of the thrust and upward displacement of the helicopter. If the collective-throttle lever is fixed, the collective pitch is reduced, the thrust force decreases, and the helicopter transitions into descent.

§ 78. Concept of Helicopter Controllability /175

Helicopter controllability refers to its ability to be rotated about its principal axes under the action of the control moments, which are created by deflection of the control command levers. The controllability is characterized by control effectiveness, control sensitivity, control lag, and the forces on the command levers.

Control effectiveness. By control effectiveness we mean the magnitude of the control moment per degree of deflection of the tilt control and per degree of change of the tail rotor pitch. Control effectiveness, and controllability as well, is divided into longitudinal, lateral, and directional. Longitudinal

control effectiveness is found from the ratio $\frac{M_z}{\eta}$ of the longitudinal control moment to the tilt control longitudinal deflection angle η. The lateral control effectiveness is defined by the ratio $\frac{M_x}{\chi}$ (where χ is the tilt control transverse deflection angle). The directional control effectiveness is found from the ratio $\frac{M_y}{\Delta\phi}$ (where $\Delta\phi$ is the tail rotor pitch change).

Longitudinal and lateral control effectiveness depends on tail rotor rpm, vertical location of the helicopter center of gravity, and horizontal hinge offset. The higher the main rotor rpm and the larger the blade thrust and centrifugal forces, the larger the longitudinal and lateral control moments and the greater the control effectiveness. For these reasons the main rotor rpm cannot be reduced markedly, since the control effectiveness decreases. The lower the helicopter center of gravity, the larger the main rotor thrust arm relative to the center of gravity and the larger the control moment and the higher the effectiveness. This means that cargo should be located as low as possible in the helicopter.

In speaking of the deflection of the tilt control relative to the universal axis, we must bear in mind that the deflection angles are severely limited and do not exceed $\eta = 4 - 6°$ in the longitudinal direction and $\chi < 4°$ in the lateral direction.

The longitudinal and lateral control effectiveness of the Mi-4 helicopter is

$$\frac{M_z}{\eta} \approx \frac{M_x}{\chi} \approx 450 \text{ kgf}\cdot\text{m/deg.}$$

<u>Control sensitivity</u>. Control sensitivity is equal to the ratio of the angular rate of rotation of the helicopter around any axis to the tilt control deflection angle. Lateral control sensitivity is usually greater than the

longitudinal and directional control sensitivities. The control sensitivity /176 is usually greater for light than for heavy helicopters. The control sensitivity depends on the control effectiveness and the damping moment. The greater the control effectiveness, the higher the sensitivity; the larger the damping moment, the lower the control sensitivity.

Control lag. The control moment which leads to rotation of the helicopter about any axis (longitudinal or transverse) is created by main rotor thrust force vector deviation. A characteristic feature of these moments is the large magnitude of the thrust force and the small magnitude of the arm of this force relative to the axis of rotation. Consequently, in order to create a control moment we must impart to the large mass of air discharged by the main rotor additional momentum in a new direction in order to obtain a new direction of the thrust force. A comparatively long time is spent on this. This time equals approximately the time for a single rotation of the main rotor and amounts to 0.2 - 0.3 seconds.

Consequently, this time is required for the helicopter to begin rotation about the longitudinal or lateral axes after the control stick is displaced. This is then the control lag. This lag will be longer, the larger the helicopter moment of inertia relative to the axis of rotation and the lower the main rotor rpm. The lag in the longitudinal control is greater than in the lateral. For comparison we take the control lag for an airplane. The airplane control moment is created by comparatively small forces with large arms. Therefore, the creation of the control moments for an airplane requires about one tenth of the time of that for a helicopter. This characteristic must be considered in helicopter piloting techniques.

Control stick force. An attempt is made to have the main rotor blades momentless. This means that with variation of the pitch the blade center of pressure shifts very little and the blade moment about its longitudinal axis scarcely changes at all. But small moment changes still arise. These variations are transmitted through the pitch control horns to the blades, from the

blade to the tilt control, and from there to the cyclic pitch control lever. High-frequency force pulsations develop on the stick and it begins to vibrate. To eliminate these vibrations, inertial or hydraulic dampers which absorb small blade oscillations are connected into the control linkage system.

In the inertial dampers the energy of the oscillatory motions is expended on rotating the pendulum, and in the hydraulic dampers on overcoming the friction forces of the piston and the fluid forced through the piston. Dampers in the control system are used on the light helicopters. Hydraulic boosters are used on the intermediate and heavy helicopters, which create mechanical /177 forces by fluid pressure on the hydraulic booster piston. These forces are used to deflect the tilt control or change the tail rotor pitch. Each control loop has its own hydraulic booster.

Irreversible boosters are most widely used at the present time; they can deflect the control organs without forces from the pilot. When moving the command lever,the pilot displaces only the slide valve piston, which regulates the fluid flowrate into the booster. Consequently, there are no forces at all on the command levers, i.e., the stick and pedals move without any resistance. This means that the pilot does not feel the helicopter control and cannot define exactly the magnitude of the command lever deflection.

In addition to the boosters, artifical feel mechanisms are provided in the helicopter control system to create definite forces on the stick to give a "control feel." These artificial feel units are provided in each control loop and consist of springs. When the command lever is deflected, one of these springs is compressed and pilot effort is expended in this compression. The magnitude of the force increases as the stick is deflected. If these forces are applied for brief periods they do not create any serious inconvenience (bearing in mind that the pilot very rarely deflects the stick fully to restore equilibrium). However, if it is necessary to alter the flight regime, for example to transition from hover to flight at maximal speed, then the stick must be moved nearly full forward and held in this position. In this

case the pilot must apply a large force to the stick for a long period. This soon causes fatigue. A trimming mechanism or a load-relieving mechanism is used to remove or regulate the force on the stick.

CHAPTER XII

HELICOPTER VIBRATIONS

§ 79. General Analysis of Vibrations /177

Periodic reciprocating motions of the elements of an elastic system can be termed vibrations or oscillations. The problem of helicopter vibrations remained unresolved for a long time; therefore, large-scale helicopter flying was not possible. Experimental flights performed prior to the middle 1940's frequently terminated in accidents as a result of severe vibrations.

Several hundred different vibrations of individual parts and of the entire helicopter as a unit can be counted on a helicopter.

Parameters of oscillatory motions. We consider an elastic plate with /178
one end clamped and a small weight on the other end (Figure 109a). If the end with the weight is deflected and then released, oscillations of the plate develop. This will be the simplest example of vibrations (Figure 109b). The oscillatory motions are characterized by three basic parameters: period, frequency, and amplitude. The period is the time for a complete oscillation (T).

Frequency is the number of periods per unit time

$$n = \frac{1}{T}.$$

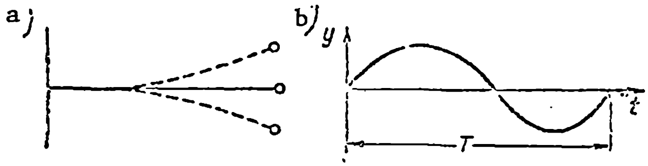

Figure 109. Parameters of vibrational motion.

Amplitude is the largest deviation of an oscillating point from the neutral position (y).

Oscillatory motion modes. With regard to nature of onset, oscillatory motions can be subdivided into forced, natural, and self-excited.

Forced vibrations are those which are caused by periodic external forces. Such forces are exciting. Forced vibrations take place with a frequency equal to that of the exciting forces. Damping forces or forces which attenuate the vibrations arise during all vibrations. The damping forces may be either internal or external. The internal damping forces arise as a result of elasticity of the material itself from which the structure is fabricated. External damping forces arise as a result of resistance of the medium in which the vibrations take place. The larger the damping forces, the faster the vibrations decay.

Natural vibrations are those which continue after termination of the action of the disturbing forces. The basic characteristic of natural vibrations is that each structure has a very definite vibration frequency, which is independent of the exciting force and is determined by the mass and stiffness of the structure.

The larger the mass of the structure, the lower the natural vibration frequency. The greater the structure stiffness, the higher the natural vibration frequency.

With regard to nature of the amplitude variation, vibrations can be divided into damped and increasing. If the amplitude decreases,in the course of time,the vibrations will be damped. Natural vibrations are always

damped. If the amplitude increases with time, the vibrations will be increasing. Increasing vibrations develop at resonance.

Resonance is coincidence of the frequency of the exciting forces with the frequency of the natural vibrations of the structure. Vibrations of helicopter parts are most often forced vibrations. /179

§ 80. Helicopter Forced Vibrations

There are in the helicopter many sources of exciting forces which cause forced vibrations. Such sources include: main and tail rotors, powerplant, transmission gearboxes, and transmission shafts.

Each of these sources creates exciting forces with a definite frequency. The lowest exciting force frequency is that of the main rotor. It may be found from the formula

$$n_{mr} = n_s k,$$

where n_{mr} is the main rotor exciting force frequency;

n_s is the main rotor rps;

k is the number of main rotor blades.

The frequency of the main rotor exciting forces varies in the range of 8 - 16 vibrations per second. The tail rotor excites forces with a frequency of 10 - 60 vibrations per second. The transmission shafts and gearboxes create a still higher frequency of the exciting forces: from 50 to several hundred vibrations per second. The powerplant yields a broad spectrum of exciting forces with frequency of 600 - 1000 vibrations per second.

The primary forced vibration source is the main rotor with hinged blade support. Blade oscillations relative to all the hinges are also the source of many vibrations.

Vibrations from the blades of the main and tail rotors are transmitted through the hubs and the airstream deflected by the blades. This slipstream strikes the tail boom and tail fin in the form of periodic pulses and causes vibrations.

All parts of the helicopter are subjected to forced vibrations, but the amplitude of these vibrations differs. The amplitude magnitude depends on the stiffness of the structure, the closeness of the source of the exciting forces, their magnitude and points of application, and on the degree of closeness to resonance. The degree of closeness to resonance is determined by the relative frequency ν, equal to the ratio of the exciting force frequency to the natural vibration frequency

$$\nu = \frac{n_{ex}}{n_{nat}}.$$

The amplitude of forced vibrations can be expressed graphically, plotting the structure deformation vertically and the relative frequency horizontally (Figure 110). The relative deformation is the ratio of the deformation caused by the dynamic load to the deformation created by the static load. From this graph we can draw the following conclusions: /180

The largest deformation or the largest amplitude occurs at resonance ($\nu = 1$). Therefore, resonant vibrations are very dangerous: they can lead to structural failure due to material fatigue;

For $\nu > 0.5$ the vibration amplitude increases very rapidly and the structural deformation increases sharply;

For $\nu > 1.5$ there is a reduction of the structural deformation in comparison with the deformation caused by a static load of the same magnitude.

Thus, to reduce the structural deformation it is necessary to reduce the degree of closeness to resonance by altering the natural vibration frequency.

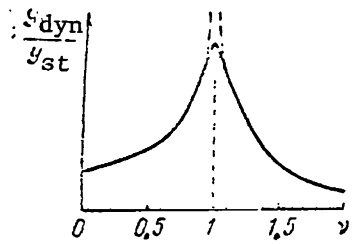

Figure 110. Relative vibration amplitude versus relative frequency.

If the exciting force frequency is high, the natural vibration frequency must be reduced. Rubber vibration dampers are used in mounting the engine to the frame to avoid resonance. The use of shock mounts reduces the stiffness of the frame-engine structure, which leads to reduction of the natural vibration frequency and increase of the relative frequency ($\nu > 1.5$).

Another example. The main rotor provides low-frequency exciting forces. The main rotor gearbox is mounted rigidly to the gearbox frame, without shock absorbers. This type of mounting increases the natural vibration frequency, and as a result the relative frequency is considerably less than 0.5.

The helicopter control linkage rods are most frequently subjected to forced vibrations. Therefore, it is particularly important to prevent resonance of the control rods. To this end the natural frequency of the rod is determined. If this frequency is close to the exciting force frequency in the region where the rod is located, the natural frequency must be changed. This frequency can be found from the approximate formula

$$n_{nat} = \frac{100D}{l^2}\sqrt{\frac{E}{\gamma}},$$

where D is the rod cross-section diameter;

l is the rod length;

E is the longitudinal elastic modulus;

γ is the specific weight of the material.

We see from this formula that the rod diameter must be increased or its length must be reduced in order to increase the natural vibration frequency.

If the rods are long, roller type supports are used to increase the frequency. When it is not possible to determine exactly the possibility of the occurrence of resonance, use is made of rods with inertial dampers. The inertial damper is a weight located inside the rod close to its midpoint, between two rubber plugs. The presence of the damper leads to rapid decay of the vibrations. /181

Under normal conditions the forced vibrations of the various parts of the helicopter are small; their amplitudes are measured in hundredths or tenths of a millimeter. However, in certain cases they may become hazardous if the normal operating conditions are exceeded.

Most frequently, magnification of the vibrations is caused by the failure of individual structural elements (stiffness is reduced and resonance occurs), by improper the adjustment of structural parts, and by mass unbalance. The acceptable vibration limit is determined by their effect on the structure and on the human organism. Vibrations are considered acceptable if they do not lead to structural failure and do not cause discomfort to the personnel (Figure 111). The higher the vibration frequency, the lower the vibration amplitude which can be endured by the personnel without pain.

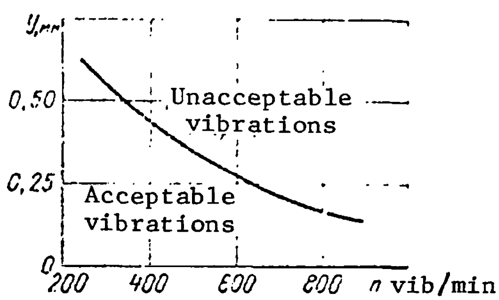

Figure 111. Acceptable vibration graph.

§ 81. Self-excited Vibrations

Self-excited vibrations are those which arise under certain conditions when constantly acting forces are transformed into periodic forces and a steady motion becomes oscillatory. To these conditions we must also add coincidence of the periodic force frequency with the natural vibration frequency.

There are three characteristic forms of self-excited vibrations in helicopters: "ground resonance," "helicopter auto-oscillations in flight," and vibrations of the flutter type.

The combination of main rotor blade oscillations relative to the vertical hinges with oscillations of the entire helicopter as it moves over the ground can be termed ground resonance. The amplitude of these oscillations increases very rapidly.

Vibrations of the ground resonance type are not observed on helicopters having main rotors without vertical hinges. In this case the blades are positioned symmetrically,and the center of gravity of the entire main rotor is located on the hub axis. As the main rotor turns the circumferential velocity of the center of gravity equals zero; consequently the main rotor centrifugal force also equals zero.

When vertical hinges are used, the blades perform oscillatory motions /182 as a result of change of the moments of the rotational drag force and the Coriolis force. Such oscillations lead to shift of the main rotor center of gravity away from the hub axis. The center of gravity begins to travel along a sinuous curvilinear trajectory (Figure 112a). The main rotor centrifugal force N appears. The appearance of this force can be explained in a different way. If the main rotor blades are positioned symmetrically relative to the hub, the resultant of the centrifugal forces of the blades taken individually equals zero. If the blades are positioned asymmetrically, the resultant of the blade centrifugal forces will be the centrifugal force of the entire main rotor.

Under the action of this force a moment is created relative to the landing gear wheel support point, which causes alternate deflection of the gear shock struts and tires (Figure 112b). Rocking of the helicopter on the gear develops. If the frequency of these (natural) oscillations coincides with the main rotor rpm, resonance occurs. The amplitude will increase

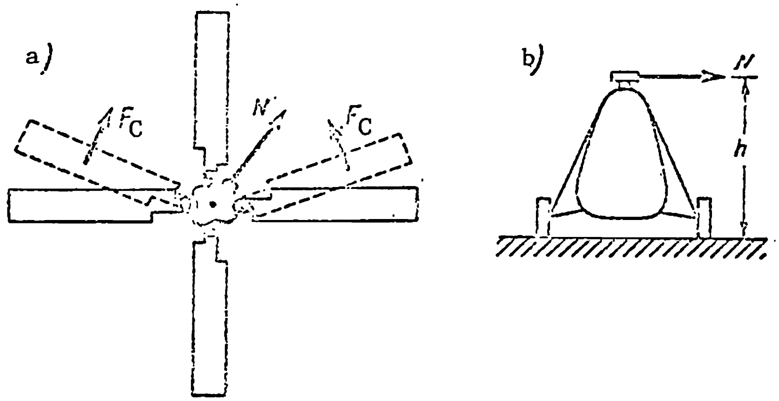

Figure 112. Occurrence of "ground resonance."

rapidly. As a result of tipping of the helicopter, moments of the blade weight forces about the vertical hinges develop. These moments amplify the oscillatory motions of the blades and lead to increase of the centrifugal force of the entire main rotor. The increase of the oscillation amplitude can lead to overturning and destruction of the helicopter. Moreover, the vibrations are amplified by the action of the gyroscopic moment of the tail rotor. The tail rotor turns at high speed and has a large gyroscopic moment. It tends to retain its axis of rotation in a fixed position. During oscillations of the helicopter the position of the tail rotor axis of rotation changes, and therefore large torsional moments develop in the fuselage tail boom and vertical fin. The actual picture of ground resonance is more complex and depends on many factors other than those examined here.

Ground resonance develops most frequently when taxiing the helicopter over rough ground, during takeoff roll and landing runout when making takeoffs /183
and landings of airplane type. But ground resonance can also occur when the helicopter is parked with the main rotor running. The basic operational causes are the following:

Low or different tension of the vertical hinge friction dampers;

Incorrect charging of the landing gear shock struts and tires.

The first factor leads to increase of the oscillatory motions of the blades and the second leads to change of the gear stiffness. As a result of the stiffness change,there is a change of the helicopter natural vibration frequency, and conditions for resonance are created.

When resonance occurs,the main rotor rpm must be decreased and the engine shut down.

Helicopter auto-oscillations in flight. Helicopter auto-oscillations are similar in nature to ground resonance. These vibrations combine oscillations of the main rotor blades relative to the vertical hinges and oscillations of the elastic elements of the helicopter fuselage. During the blade oscillations a centrifugal force of the main rotor arises,and this leads to whipping of the shaft and deformation of the members of the frame supporting the gearbox and the structural elements of the fuselage.

Auto-oscillations occur very rarely on single-rotor helicopters; only in case of failure of individual structural elements of the fuselage, which results in reduction of the fuselage stiffness, or when the vertical hinge dampers are out of adjustment. These vibrations are observed more frequently in the dual-rotor helicopters with tandem arrangement of the lifting rotors. The bending stiffness of the fuselage of this helicopter in the horizontal plane is comparatively low. If there are large oscillations of the blades of both rotors, large bending moments are created, which as they change their direction cause marked bending vibrations of the fuselage.

Rotor blade flutter. Vibrations of the flutter type are the most hazardous. They are encountered on the main rotor blades and are theoretically possible on the tail rotor blades, but in view of the high stiffness of the latter they are not encountered in practice.

Main rotor blade flutter may be of two types: bending-torsion and flapping. Bending-torsion flutter in the pure form is observed most frequently for blades with rigid restraint at the hub. Blades with hinged support usually show the combined type of flutter.

§ 82. Bending and Bending-Torsion Vibrations of Rigidly Restrained Blade

Bending vibrations are those in which the blade chord displaces parallel to itself, i.e., there is not twisting of the blade. These vibrations occur when the line of blade section centers-of-gravity coincides with the elastic axis. If the tip of such a blade is bent down, an elastic force P_{el} appears, applied at the elastic center and directed upward (Figure 113). When the deflected blade is released, it begins to travel upward with an acceleration under the action of the elastic force. As a result of the acceleration there is formed the inertia force P_{in}, directed downward and applied at the center of gravity. We are examining a blade for which these points coincide; therefore the twisting moment of the inertia force will be zero, and there will be no twisting of the blade. /184

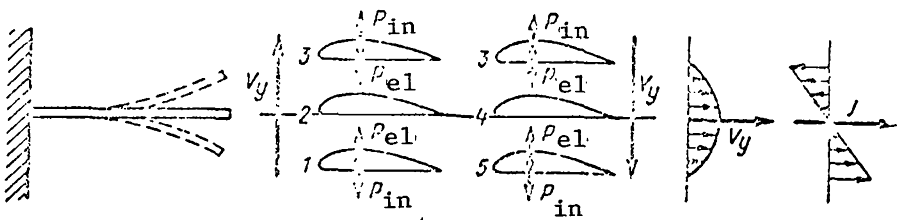

Figure 113. Blade bending vibrations.

As the blade element moves upward, the elastic and inertia forces will decrease, while the bending motion velocity V_y will increase. When the blade element reaches the neutral position, the elastic and inertia forces will be equal to zero while the velocity will be maximal. With further upward

movement of the blade,the elastic force changes its direction and will cause reduction of the velocity. The element motion stops in the extreme upper position. After passage through the line of equilibrium, the inertia force will be directed upward. This motion is shown in Figure 113, positions 1, 2. 3. The blade element will travel downward from position 3 (positions 3, 4, 5). The forces, acceleration, and velocity will vary just as in the upward motion. It follows from this analysis that the largest elastic forces and the largest accelerations of the bending motion will occur in the extreme positions. The inertia forces will be maximal for the largest accelerations, i.e., also in the extreme positions. If the blade section passes through the neutral position, the forces and acceleration change their signs and the velocity reaches its maximal magnitude. The variation of the velocity and acceleration is shown on the graphs of the vibratory motion. If the line of centers-of-gravity does not coincide with the elastic axis, the inertia force of the given blade element creates a torsional moment, which twists the blade relative to the elastic axis. The bending vibrations will be supplemented by torsional vibrations.

For most blades the line of centers-of-gravity lies behind the elastic axis. The nature of the bending-torsion vibrations of such a blade is shown in Figure 114. The direction and variation of the forces, velocity, and acceleration will be the same as for bending vibrations. As a result of the aft shift of the center of gravity relative to the elastic center, in the case of upward bending motion of the blade section there are created the additional positive twist angles ($+ \phi$), which increase up to the neutral /185
position and then decrease (positions 1 - 5).

During downward bending motion of the blade, negative twist angles are created, which first increase in absolute magnitude and then decrease (positions 5 - 9).

This nature of the natural bending-torsion vibrations is determined by the mutual position of the elastic axis and the line of centers-of-gravity, and occurs for all blades.

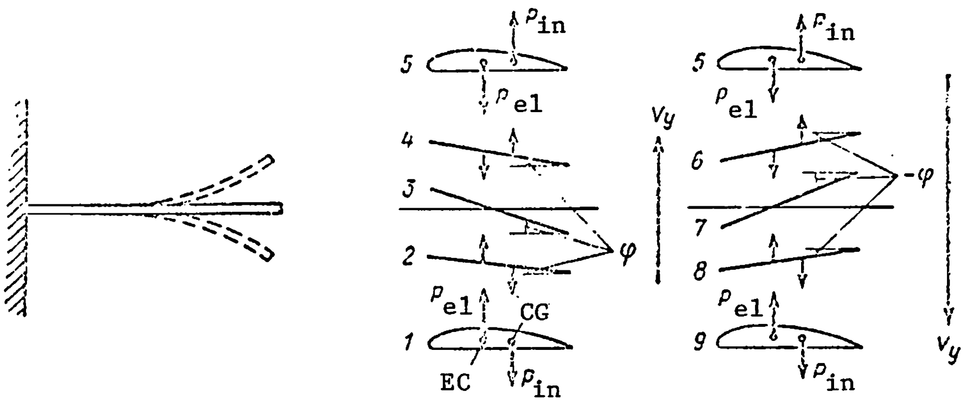

Figure 114. Blade bending-torsion vibrations.

We have examined bending and bending-torsion vibrations of the blade in stationary air, i.e., when the rotor is not turning. If the rotor turns, the aerodynamic forces must be added to the elastic and inertia forces. The effect of these forces on the blade leads to flutter under certain conditions.

Essence of bending-torsion blade flutter. We imagine that bending-torsion vibrations of a blade take place while the main rotor turns. We resolve these vibrations along the blade line of motion with the air flow approaching at the velocity u (Figure 115). The elastic and inertia forces which arise during bending-torsion vibrations are not shown in Figure 115, but they do act. To these forces we add the additional lift force ΔY. As the blade flexes upward the twist angles will be positive, and this means that /186
there will be a positive increment ($\Delta a = \phi$) of the blade element angle of attack.

The additional lift force will be directed upward. When the flexing motion is downward, the twist angles will be negative, which means there will be a negative angle of attack increment ($- \Delta a = - \phi$). The additional lift force will be directed downward. It follows from this analysis that the additional lift force coincides with the direction of the bending motion, i.e., it will be an exciting force.

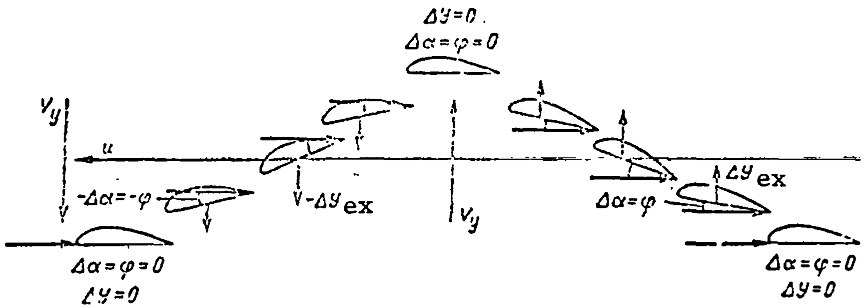

Figure 115. Blade bending-torsion flutter.

Under the influence of the exciting force the bending vibration amplitude will increase, which leads to increase of the elastic forces, bending vibration accelerations, inertia forces, and their torsional moment. The twist angles and the additional lift forces increase. The conclusion is that bending vibrations cause twisting of the blade. Increase of the twist leads to increase of the bending vibrations, which in turn leads to increase of the torsional vibrations.

As a result the amplitude of the vibrations increases so rapidly that blade failure may occur.

Flutter occurrence conditions. Both exciting forces and damping forces arise in the case of bending-torsion vibrations. Flutter will be possible only if the exciting forces are larger than the damping forces. We have already shown that the exciting forces arise as a result of torsional vibrations, which lead to increase of the angle of attack.

$$\Delta Y_{ex} = \Delta C_y S_b \frac{\rho u^2}{2}. \tag{49}$$

The lift force coefficient increment ΔC_y can be found from the plot of $C_y = f(a)$ (Figure 116a).

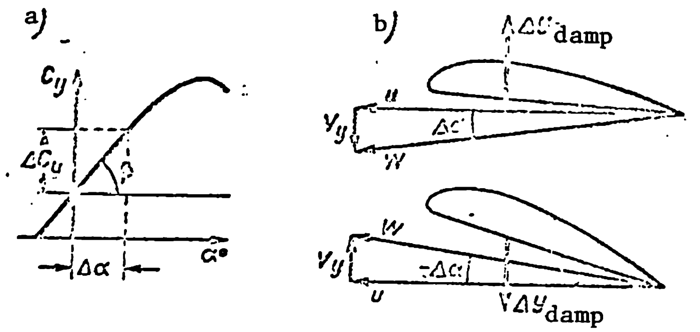

Figure 116. Creation of damping force.

We see from the figure that the increment ΔC_y depends on the angle of attack increment $\Delta\alpha$ and on the slope of the curve to the abscissa axis, which can be defined as $\operatorname{tg}\beta = \frac{\Delta C_y}{\Delta\alpha} = a$. Hence

$$\Delta C_y = a\Delta\alpha = a\varphi.$$

Substituting the value of ΔC_y into (49), we obtain /187

$$\Delta Y_{ex} = a\varphi S_b \frac{\rho u^2}{2}.$$

Thus, the exciting forces are proportional to the twist angle and the square of the blade tip circumferential velocity.

The damping forces arise as a result of the bending vibrations. During these vibrations there is a change of the blade element angle of attack as a result of the vertical velocity V_y of the bending motions (Figure 116b). In the case of downward bending motion, the angle of attack increment is positive and the additional lift force is directed upward. In the case of downward bending motion, $\Delta\alpha$ is negative and the additional lift force is directed downward. Thus, in all cases of bending motion the additional lift force is directed opposite this motion, i.e., it is a damping force

$$\Delta Y_{damp} = \Delta C_y S_b \frac{\rho u^2}{2}. \tag{50}$$

The lift force coefficient increase is proportional to the angle of attack, i.e., $\Delta C_y = a\Delta\alpha$.

From Figure 116b $\Delta\alpha = \frac{V_y}{u}$; therefore,

$$\Delta C_y = a\frac{V_y}{u}.$$

Substituting the value of ΔC_y into (50), we obtain

$$\Delta Y_{damp} = aV_y S_b \frac{1}{2}\cdot\rho u.$$

The conclusion is that the damping force is proportional to the velocity of the bending motion and the blade tip circumferential velocity. This is shown graphically in Figure 117, from which we see that the damping forces equal the exciting forces at a definite circumferential velocity. This velocity is called the critical flutter speed. Since $u = 2\pi Rn_s$, the critical speed corresponds to the critical flutter rpm.

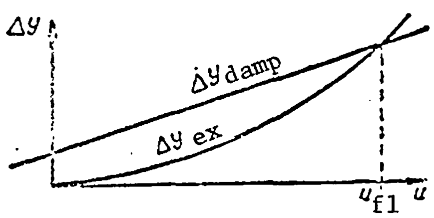

Figure 117. Exciting and damping forces versus speed.

At speeds below the critical flutter speed (for rpm below the critical value) the damping forces are larger than the exciting forces, the bending-torsion oscillations will be damped, and flutter is not possible. At rpm above the critical value the damping forces are smaller than the exciting forces. The bending-torsion vibrations will be increasing and flutter is inevitable. Consequently, flutter develops if the blade section cg line is behind the elastic axis /188
and the main rotor rpm is above the critical flutter rpm. The first of these conditions holds for all blades; therefore the second condition is sufficient for the onset of flutter.

Dependence of critical flutter rpm on various conditions. The critical flutter rpm depends on the blade stiffness, mutual positioning of the cg line and the elastic axis, mutual positioning of the center-of-pressure line

and the elastic axis. The higher the blade stiffness in bending and torsion, the larger the exciting forces must be in order to deform the blade.

High velocity is necessary in order to obtain large forces. This means that the critical flutter rpm increases with increase of the stiffness. Flutter of a metal blade is possible at a considerably higher rpm than for a blade of composite construction of the same dimensions. This is one of the advantages of metal blades.

If the blade cg axis coincides with the elastic axis, the natural vibrations will be flexural; without the torsional vibrations there will not be any exciting forces, and flutter will not occur.

If the cg axis is ahead of the elastic axis, the natural vibrations will be the bending-torsion type. However, the signs of the twist angles will be opposite to those for which flutter occurs, i.e., when the blade flexes upward the twist angles will be negative,and when the blade bends down the angles will be positive. With these vibrations associated with twisting, additional damping forces are created and flutter is impossible.

If the cg axis is behind the elastic axis, then the larger the distance between these axes,the larger the twisting moment of the inertia forces, and the lower the critical flutter rpm. This means that the blade cg axis must be moved forward in order to increase the critical rpm. To this end special weights in the form of metal bars, mass balance weights, are installed in the leading edge of the blade.

The mutual influence of the center-of-pressure axis and the elastic axis amounts to the fact that if the center-of-pressure axis is ahead of the elastic axis,an additional twisting moment on the blade is created from the action of the aerodynamic forces. The sign of this moment is the same as the sign of the inertia force moment. Consequently, in this case the critical flutter rpm will decrease.

Flutter characteristics of blade with hinged support. The majority of main rotors have blades with hinged support. The presence of the horizontal and axial hinges leads to the so-called flapping flutter, i.e., a combination of flapping motions with oscillations of the blade relative to the axial hinge. The amplitude of such oscillations will increase. These oscillations are possible even for an absolutely rigid blade, in the absence of bending and twist. /189

During the flapping motions there is an increase of the amplitude of the vertical displacements of the blade element. The amplitude is made up of the flapping and bending displacements. In this case the critical flutter rpm decreases. Consequently, the hub horizontal hinges contribute to the onset of flutter.

The axial hinges permit the blade to rotate about its longitudinal axis. This is equivalent to reduction of the blade torsional stiffness. Rotation about the axial hinge is possible as a result of elastic deformation of the blade pitch control linkage elements. The lower the stiffness of these elements, the lower the critical flutter rpm. The hub axial hinges also promote the onset of flutter.

The flapping compensator causes reduction of blade pitch as they flap upward, and increase of the pitch as they flap downward. This pitch change is analogous to the bending-torsion oscillations during flutter. Consequently, the flapping compensator also promotes the onset of flutter. The larger the compensation coefficient, the lower the critical flutter rpm. For most hubs the compensation coefficient $K = 0.5$. In the case of a large value of this coefficient, the critical flutter coefficient decreases to rpm values within the operational range.

The critical flutter rpm depends on the flight speed: with increase of the speed there is an increase of the resultant velocity of the flow over the blade at the 90° azimuth, and this promotes the onset of flutter. This means that the critical flutter rpm decreases with increase of the flight speed.

In addition to the factors listed above, blade flutter is affected by centrifugal forces. As a result of the centrifugal forces the flapping motions and bending of the blade are reduced, and the blade becomes effectively stiffer. Consequently, the critical flutter rpm increases under the influence of the centrifugal forces.

In view of the lower blade stiffness, flutter of the main rotor blades does not develop as violently as does the flutter of the airplane wing, and therefore rotor blade flutter can be detected in time and measures can be taken to stop the flutter.

Operational sources of flutter. Flutter is avoided in the design of airplanes and helicopters. This means that in calculating the critical flutter rpm the blade stiffness and cg location are selected so that the critical flutter rpm is made considerably higher than the maximal permissible main rotor rpm. However, flutter can arise from operational causes, as a result of mass unbalance and reduction of the structural stiffness. Disruption of /190
the mass balance is particularly characteristic for blades of composite skeleton construction. In these blades the wooden ribs absorb moisture from the air markedly. With increase of the moisture content, the cg line shifts aft, which leads to reduction of the critical flutter rpm, and flutter becomes possible at operational rpm.

The mass balance may be disrupted as a result of careless overhaul of the blade, which also leads to the onset of flutter. Reduction of the structural stiffness, which leads to reduction of the critical flutter rpm, occurs if there is a failure of the individual structural elements or in case of damage to the blade skin.

Under operational conditions flutter may occur with the main rotor operating on the ground and in flight. Flutter is detected by heavy vibration of the helicopter and from "blurring" of the main rotor cone of rotation. If there are no blade vibrations, the blades of a properly adjusted rotor will

travel along a single track and will form a definite cone, which is visible from the cockpit. If vibrations are present, the blades travel along different trajectories and the cone will be "blurred," vague.

When flutter is detected, the main rotor rpm must be immediately reduced to the minimal permissible value. After landing the reason for the flutter must be investigated. It must be kept in mind that "blurring" of the cone occurs not only in the case of flutter, but also if the blades are not in track, i.e., in case of improper adjustment of the main rotor. However, in the latter case the "blurring" is independent of the rpm.

Measures to prevent vibrations of all types. During helicopter operation, special attention is devoted to preventing all forms of vibration. These measures concern, first of all, strict adherence to all the operating conditions and performance of all the instructions and scheduled maintenance procedures for the particular helicopter type. These operations include the adjustment and alignment of all parts of the helicopter, verifying the proper adjustment of the vertical hinge dampers, proper charging of the landing gear shock absorbers and tires, verifying main rotor tracking, and checking the main rotor for flutter.

The essence of the flutter check amounts to the following: a definite weight is attached to the trailing edge of each blade and the main rotor rpm is increased (helicopter parked) to the rpm indicated in the instructions. Then the rpm is increased by 1 - 2%, and the rotor is operated in this condition for 1 - 2 minutes. Then the rpm is again increased and brought up to the maximal value indicated in the instructions. If flutter does not occur with the weights installed, there will be just that much more margin with the weights removed, since the critical flutter rpm increases as the blade cg moves forward.

APPENDIX I

SYMBOL LIST

A — main rotor angle of attack

α — blade element angle of attack

T — main rotor thrust force

T_0 — thrust force at altitude $H = 0$

T_H — thrust force at altitude $H > 0$

T_y — part of main rotor thrust force directed along hub axis

Y — main rotor lift force

P — main rotor propuslive force

P_x — main rotor drag force

S_s — side component of main rotor thrust

P_{sp} — main rotor specific thrust

ΔT — blade element thrust

Q_b — blade rotational drag force

ΔQ — elemental rotational drag force

H — longitudinal component of main rotor thrust force

F_C — blade Coriolis force

N — blade centrifugal force

X_{par} — helicopter parasite drag

N_{avail} — power available

N_e — effective engine power

N_{req} — power required

N_h — power required for horizontal flight

N_{hov} — power required for hovering

N_{cl} — power required for climb

N_{des} — power required for descent with engine operating

N_m — power required for motion

N_i — induced component of power required

N_{pr} — profile component of power required

M_{tor} — torsional moment

M_r — reactive moment of main rotor

$M_{r_{t.r}}$ — reactive moment of tail rotor

$M_{t_{t.r}}$ — thrust moment of tail rotor

M_z — helicopter pitching moment

M_x — helicopter rolling moment

M_y — helicopter yawing moment

C_t — main rotor thrust coefficient

C_y — blade element lift force coefficient

μ — main rotor operating regime coefficient

η_0 — main rotor relative efficiency

m_{tor} — torsional moment coefficient

m_{tor_i} — induced drag torsional moment coefficient

$m_{tor_{pr}}$ — profile drag torsional moment coefficient

χ — main rotor end loss coefficient

ζ — power utilization coefficient

D — main rotor diameter

R — main rotor radius

r — blade element radius

$\bar{r}$ — blade element relative radius

F — area swept by main rotor

k — number of blades

b — blade element chord length

G — helicopter weight

G_b — main rotor blade weight

x_{cg} — helicopter horizontal cg location

y_{cg} — helicopter vertical cg location

ρ — air density

Δ — relative air density

ϕ — blade element incidence angle (pitch)

$\phi_{0.7}$ — main rotor incidence angle (pitch)

β — blade flapping angle

ψ — blade azimuth angle

α_0 — main rotor coning angle

a_1 — main rotor coning axis fore-and-aft tilt

b_1 — main rotor coning axis lateral tilt

V — helicopter flight velocity

V_i — induced velocity (inflow velocity)

u — blade tip circumferential velocity

W — blade element resultant velocity

V_{des} — vertical descent velocity

ω — main rotor angular velocity

APPENDIX II

PROGRAMMED TESTING ANSWERS

Chapter 3. Main Rotor Operation in Vertical Flight Regime

Question 1.	Correct answer 2.	Question 5.	Correct answer 2.
Question 2.	Correct answer 4.	Question 6.	Correct answer 2.
Question 3.	Correct answer 2.	Question 7.	Correct answer 1.
Question 4.	Correct answer 1.	Question 8.	Correct answer 1.

Chapter 4. Main Rotor Operation in Forward Flight Regime

Question 1.	Correct answer 1.	Question 5.	Correct answer 3.
Question 2.	Correct answer 3.	Question 6.	Correct answer 2.
Question 3.	Correct answer 2.	Question 7.	Correct answer 1.
Question 4.	Correct answer 1.		

Chapter 5. Helicopter Vertical Flight Regime

Question 1.	Correct answer 1.	Question 3.	Correct answer 2.
Question 2.	Correct answer 3.	Question 4.	Correct answer 1.

Chapter 6. Helicopter Horizontal Flight

Question 1.	Correct answer 3.	Question 3.	Correct answer 3.
Question 2.	Correct answer 2.	Question 4.	Correct answer 1.
		Question 5.	Correct answer 3.

Chapters 7 and 8. Helicopter Climb and Descent Along Inclined Trajectory

Question 1.	Correct answer 1.	Question 4.	Correct answer 2.
Question 2.	Correct answer 1.	Question 5.	Correct answer 1.
Question 3.	Correct answer 3.		

Chapter 9. Helicopter Flight in Main Rotor Autorotative Regime

Question 1.	Correct answer 3.	Question 4.	Correct answer 3.
Question 2.	Correct answer 1.	Question 5.	Correct answer 2.
Question 3.	Correct answer 1.		

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